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Band-limited feedback cancellation with a modified filtered-X LMS algorithm for hearing aids

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Abstract

Commonly used wideband adaptive feedback cancellation techniques do not provide satisfactory performance for reducing feedback oscillation in hearing aids. In this paper, a band-limited adaptive feedback cancellation algorithm using normalized filtered-X LMS techniques is proposed that provides good cancellation efficiency, convergence behavior and better output sound quality for speech signals, when compared to the wideband approach. Convergence analysis and computer simulations illustrate the advantages of the proposed approach.

Keywords: Hearing aids; Acoustic feedback cancellation; Adaptive filtering; Oscillation frequency; Filtered-X LMS; Convergence behavior

1. Introduction

Hearing aids amplify incoming sound so as to increase the signal level for the hearing-aid user. Due to imperfect earmold fitting and venting in the hearing aid device, there is acoustic leakage from the receiver to the microphone. The leakage causes a regenerative feedback loop, which frequently makes the hearing aid oscillate and results in a whistling sound. For people with severe hearing loss, this problem is serious because at high gains there is an increased risk of squealing which may render the hearing aid useless. Feedback becomes more apparent when an object is close to the ear (i.e. when using a telephone) or when the jaw is moving (i.e. when chewing). In case of squealing, hearing aid users tune down the hearing aid gain or fit hearing aids more tightly in the ear canal, but such adjustments may compromise the function and comfort of the hearing aid.

Techniques for better earmold venting, coupling of hearing aids, and fitting methods have been proposed to reduce feedback problems (Cox, 1982; Dillon, 1991). In addition to approaches that optimize hearing aid devices, signal processing techniques can be used to reduce feedback (Agnew, 1993; Nielsen and Svensson, 1995; Svensson, 1995; Bustamante et al., 1989; Engebretson et al., 1990; Dyrlund and Bisgaard, 1991; Graupe et al., 1988; Kates, 1991; Maxwell and Zurek, 1995; Joson et al., 1993). Among them, adaptive feedback cancellation is the most attractive approach since it can provide a higher maximum stable gain than other techniques.

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Most of the reported adaptive feedback cancellation methods use wideband approaches. However, the signal from the external sound source acts as an interference signal for the adaptive filter. A bias will be introduced in the feedback path estimation if the interference signal is correlated with the feedback canceller input (Siqueira et al., 1997; Siqueira and Alwan, 1999). Since the feedback canceller is adaptive, the non-zero interference signal will also introduce a noisy coefficient adaptation which results in distortion at the hearing aid output.

Many approaches have been proposed to reduce the influence of signals from external sound sources on the feedback cancellers' adaptation. For example, probe noises insert broadband noises into hearing aids to provide strong excitation signals for adaptive filtering (Engebretson et al., 1990; Dyrlund and Bisgaard, 1991; Graupe et al., 1988). The injection of probe noises must be carefully controlled, otherwise the output sound will be contaminated by the added noise.

Another category of feedback cancellers employs non-continuous adaptation schemes for adaptive filters. In the system proposed by Graupe et al. (1988), the forward signal path is disconnected, and the adaptive filter deals with an internal broadband probe noise, at system turn-on, periodically, and/or when a certain gain change is detected. Kates (1991) proposed a system in which a second-order adaptive notch filter is used to detect oscillation. The adaptation of the feedback canceller is enabled only after a strong oscillation is detected. In (Maxwell and Zurek, 1995) adaptation and probe noise insertion are activated when the level of the processed signal is below a predefined threshold (a "quite period"), or when strong oscillation is detected. Because of non-continuous adaptation, the stable gain improvement of the feedback cancellation algorithms can be limited.

The above-mentioned methods adopt wideband feedback cancellation approaches. Distortion still exists in the hearing aid outputs even though the methods do increase the stable gain of the devices. In addition, oscillation is not suppressed completely.

In this paper, acoustic feedback and the fundamental problems of wideband feedback cancellation are investigated. Utilizing the characteristics of the feedback path, we develop a band-limited adaptive feedback canceller with a modified normalized filtered-X LMS algorithm (Chi et al., 1999; Chi, 1999), which has better cancellation efficiency, convergence behavior, and better output sound quality than the wideband algorithms when the input is speech. In the study of Kates (2001), Hellgren (2000), Hellgren and Forssel (2000), filtered-X LMS algorithms are also used for feedback cancellation. In addition, in Hellgren's work, there is a pre-filter which attempts to whiten the input signal and thus reduce the bias.

In the following subsections, we discuss how acoustic feedback is generated, common acoustic feedback cancellation (AFC) techniques, and the limitation of wideband AFC approaches. In Section 2, we introduce a new AFC technique, and an analysis of its convergence is in Section 3. Sections 4 and 5 present computer simulation and subjective evaluation results of the AFC algorithms, respectively. A summary is presented in Section 6.

1.1. Acoustic feedback in hearing aids

In the most basic form, a hearing aid device is composed of a microphone, an amplifier circuit, and a receiver. Acoustic leakage and mechanical coupling form a closed-loop system. Since the signal processing functions are only applied to electrical signals, we only consider the signal path from the microphone output to the receiver input. A hearing aid model is shown in Fig. 1. In the block diagram, double lines and single lines represent acoustic paths and electrical paths, respectively. $s(n)$ is the microphone output signal which comes only from the external sound source. $q(n)$ is

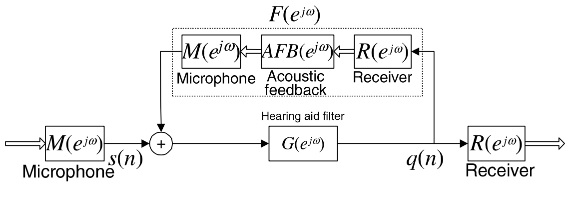

Fig. 1. Block diagram for a hearing aid model.

the receiver input signal. The feedback path $F(e^{j\omega})$ comprises the electroacoustic transfer function of the receiver $R(e^{j\omega})$, the acoustic feedback path transfer function $AFB(e^{j\omega})$, and the acousto-electric transfer function of the microphone $M(e^{j\omega})$. $G(e^{j\omega})$ is the transfer function of the amplifier circuit. Throughout this paper, we will refer to the feedback path by $F(e^{j\omega})$. Without feedback, the hearing aid functions as an acoustic transfer function which comprises $M(e^{j\omega})$, $G(e^{j\omega})$ and $R(e^{j\omega})$ and usually has gain greater than 0 dB in some frequency bands for hearing loss compensation. With feedback, the amplified sound at the receiver output is propagated back to the microphone and re-amplified. As a result, the hearing aid functions as a closed-loop acoustic transfer function $M(e^{j\omega})G(e^{j\omega})R(e^{j\omega})/(1-G(e^{j\omega})F(e^{j\omega}))$, where $F(e^{j\omega}) = M(e^{j\omega})AFB(e^{j\omega})R(e^{j\omega})$. Instability of this system may cause a hearing aid to oscillate at some frequencies. These oscillation components result in a whistle which may have a narrow bandwidth and a prominent peak in its spectrum. Oscillation occurs at those frequencies, ω_{osc}, where the open-loop transfer function $G(e^{j\omega_{osc}})F(e^{j\omega_{osc}})$ meets the Nyquist criteria (Nyquist, 1932; Egolf, 1982),

1. $|G(e^{j\omega_{osc}})F(e^{j\omega_{osc}})| \geqslant 1$;
2. $\angle G(e^{j\omega_{osc}})F(e^{j\omega_{osc}}) = m \times 360°$, where m is integer.

There are some important facts about the oscillation. First, the oscillation does not depend on the characteristics of the incoming signal. Second, a large hearing aid gain may produce more oscillation frequencies while resulting in higher build-up rates. Third, if the delay in the forward path increases, the number of oscillation frequencies may increase as well.

In addition to oscillation, a hearing aid may produce unpleasant sound when it is in a sub-oscillatory state. Sub-oscillation occurs when the above in-phase condition is met and the magnitude of the hearing aid open-loop transfer function is less than but close to unity. Both oscillation and sub-oscillation should be eliminated so that hearing aids are able to offer sufficient amplification without deteriorating sound quality.

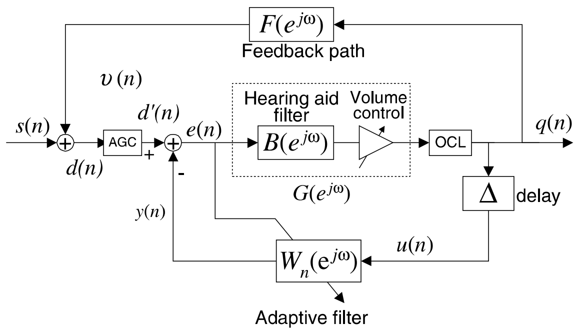

Fig. 2. Block diagram for adaptive feedback cancellation.

1.2. Adaptive feedback cancellation in hearing aids

The configuration of a common adaptive feedback cancellation scheme is shown in Fig. 2, where the amplification transfer function $G(e^{j\omega})$ is composed of the hearing aid filter $B(e^{j\omega})$ and volume control. Automatic gain control (AGC) and output compression limiting (OCL) are applied to the signal at the microphone ($d(n)$) and the output of the hearing aid processing unit, respectively. An adaptive filter $W_n(e^{j\omega})$ tracks the time-varying feedback path and cancels the feedback signal from the hearing aid input. If a shorter adaptive filter is used to reduce complexity, a delay Δ is inserted so that the adaptive filter can cover the most significant part of the feedback path's impulse response. Due to the low computation requirement for a hearing aid, simple stochastic-gradient based algorithms are commonly used. To provide good feedback cancellation, the convergence of adaptive filter must be fast enough to track possible acoustic feedback change and/or the AGC gain change.

1.3. Fundamental problems of wideband feedback cancellation

For the topology of the feedback canceller shown in Fig. 2, the signal $s(n)$, which is from the external sound source, acts as an interference signal for the feedback path estimation. Using the mean-squares-error criterion, a bias will be introduced in the feedback path estimation by the adaptive filter when $s(n)$ is not white and modeled by the auto-regressive model (Siqueira et al., 1997; Siqueira and Alwan, 1999), which is typically used to model speech. If the bias is significant, we do not expect good cancellation efficiency. Moreover, the feedback canceller is an adaptive system whose adaptation is influenced by the error signal $e(n)$, which is typically non-zero. Therefore, the impulse response of the feedback canceller changes at each time instance. This noisy deviation of the adaptive filter coefficients from steady-state values is typically called the "misadjustment". Using the coherence function to characterize this non-linear distortion, we showed that the distortion caused by coefficient variation is significant when the bias and coefficient variation are large, and that bias is significant in the frequency band where most of the external signal $s(n)$ is concentrated (Chapter 2, Chi, 1999). For speech, distortion would be most apparent at low frequencies where the signal energy is concentrated.

In addition to the aforementioned problems, the adaptive feedback canceller can track oscillation components well when the energy of the oscillation components (the signal to be suppressed) in the adaptive filter input and error are comparable to the energy of the signal from the external sound source (acting as an interference noise) (Gunnarsson and Ljung, 1989). If $s(n)$ has significant energy in the bands where oscillation frequencies are not located, the oscillation suppression ability of the wideband feedback canceller will be compromised.

2. A new feedback canceller

The major task of the adaptive feedback canceller is to eliminate oscillation. Therefore, the adaptive filter only needs to approximate the feedback path at or near the oscillation frequencies. Based on the Nyquist criteria described earlier, oscillation only occurs at a limited number of frequencies at which both magnitude and phase criteria are met. For a hearing aid application, the oscillation frequencies are typically located in the frequency regions where high gain amplifications are required to compensate for the hearing loss. Based on our experience working with patients

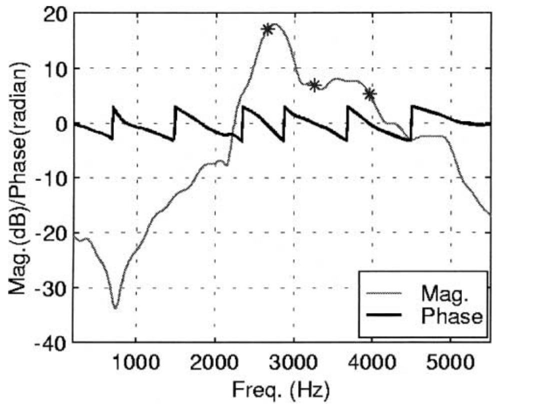

Fig. 3. Example of an open-loop transfer function (magnitude and phase) and oscillation points measured on KEMAR with a prototype ITE digital hearing aid programmed for a typical high frequency hearing loss.

with high frequency hearing loss and ITE hearing aids, the oscillation frequencies are mostly above 2000 Hz. Fig. 3 shows an example of a hearing aid open-loop transfer function measured on KEMAR ear using a prototype ITE digital hearing aid programmed for a typical high frequency hearing loss. In the figure, asterisks denote oscillation points.

It was our hypothesis that if the adaptive filter is constrained to operate only in the regions that are known to contain oscillation frequencies, then the filter would be more efficient in suppressing oscillation, and introduces less distortion than the wideband approach.

Fig. 4 shows the proposed band-limited adaptive feedback canceller. The input and error sam-

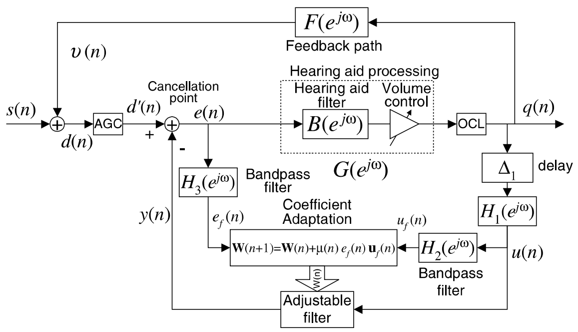

Fig. 4. Block diagram of the band-limited adaptive feedback canceller.

ples ($u(n)$ and $e(n)$) are band-limited by the filters $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$, respectively, before being used to adapt the weights of the adaptive filter. These filters are chosen to preserve all possible oscillation frequencies and remove other signal components so that the adaptive filter only responds to the signals at or near oscillation frequencies. $H_1(e^{j\omega})$ is used to band-limit the feedback cancellation signal to the frequency region which contains oscillation frequencies. By doing so, the non-linear distortions associated with the adaptive digital filter are limited to a reduced frequency band. A delay Δ_1 is inserted so that the adaptive filter can cover the most significant part of the feedback path's impulse response.

For speech inputs, this idea is attractive since most of the energy in speech signals is at low frequencies, while oscillation normally occurs at high frequencies. The magnitude of spectral peaks of the band-limited signals $(u_f(n), e_f(n))$ would be lower than those of the original signals ($u(n)$ and $e(n)$). The feedback canceller can better target oscillation components so that the oscillation suppression efficiency is increased. Furthermore, because the error signal energy is reduced, the misadjustment of the adaptive feedback canceller is reduced significantly when compared to the performance of a wideband approach.

2.1. A normalized filtered-X LMS algorithm

To satisfy the low computational constraint for a hearing aid and perform band-limited adaptive feedback cancellation, we adopt a normalized filtered-X LMS adaptive filtering algorithm (Widrow and Stearns, 1985). The M-tap coefficient-vector $\boldsymbol{W}(n)$ is adapted as

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \mu(n)e_f(n)\boldsymbol{u}_f(n),$$

where $\boldsymbol{u}_f(n) = u_f(n), (u_f(n-1), \ldots, u_f(n-M+1))^{\mathrm{T}}$, and $u_f(n)$ is the output of $H_2(e^{j\omega})$, $e_f(n)$ is the output of $H_3(e^{j\omega})$, $\mu(n)$ is the normalized step-size.

With this filtered-X LMS algorithm, the adaptive filter operates in the band of interest to accomplish the desired band-limited adaptation and filtering (Chi et al., 1999; Chi, 1999). The bandpass filter $H_2(e^{j\omega})$ provides proper time-alignment for LMS adaptation. There is a constraint on the phase responses of the bandpass filters $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$. This phase constraint will be derived in Section 3.2. For simplicity, we select two identical bandpass filters for $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$.

When a speech input signal is applied, a burst in the error signal $e(n)$ mostly originates from the external sound source. When this happens, the adaptive step-size of the feedback canceller should be reduced. Therefore, the power estimation calculation should instantaneously respond to the level of the current adaptive filter input and error signals to provide a proper adaptation step-size at the onset of a burst in a speech signal. We include the error signal samples in the power calculation ($p(n)$).

$$p(n) = \rho p(n-1) + u_f^2(n) + e_f^2(n),$$

$$\mu(n) = \frac{\gamma}{p(n) + c},$$

where ρ is the forgetting factor for the power estimation, c is a constant to prevent singularities, and γ is the step-size parameter. With this minor modification, our adaptive filter is more dedicated to the feedback cancellation problem than the conventional normalized algorithms by providing a proper adaptation step-size especially at the onset of bursts.

2.2. Identification of the oscillation frequencies

In order to provide efficient band-limited feedback cancellation, we have to determine the oscillation frequencies so that the bandwidths of the bandpass filters can be specified. To find oscillation frequencies, we first set the three filters ($H_1(e^{j\omega})$, $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$) in Fig. 4 to be all-pass. Then we disconnect the hearing aid filter and inject a pseudo-random noise signal internally. The coefficients of the adaptive wideband feedback canceller are then used to obtain information about the open-loop transfer function. Applying the Nyquist criterion for feedback instability to the open-loop transfer function, we calculate the oscillation frequencies. After identifying the oscillation frequencies, we can design the bandpass filters so that the pass-band of the filter only covers frequencies at and near the oscillation frequencies. Since the acoustic feedback characteristics vary with different hearing aids and wearers, the aforementioned procedure is performed for each individual user.

2.3. Filter selection

Two important points should be taken into consideration when choosing the filters. First, in order to suppress oscillation, the adaptive filter must generate an output signal which will cancel the feedback signal, especially at the oscillation frequencies. If the delay introduced by $H_1(e^{j\omega})$ is too long, the signal generated by the adaptive filter may lag behind the feedback signal and fail to cancel it. On the other hand, if a short adaptive filter is used to reduce complexity, we can add a delay in front of the bandpass filter so that the adaptive filter can cover the most significant part of the feedback path's impulse response. Second, when the insertion gain of the hearing aid increases, the frequency region which satisfies the first Nyquist criterion expands. Accordingly, the bandwidth of the bandpass filters would need to increase. However, a broader bandwidth will compromise the improvement which comes with the band-limited adaptive feedback cancellation approach. In a practical implementation, we pre-calculate several sets of bandpass filter coefficients for different volume control settings. The appropriate set of coefficients is selected according to the hearing aid gain setting. In the proposed method, a strict accuracy requirement for the magnitude responses of the bandpass filters at the oscillation frequencies is not critical for good feedback cancellation. Typically, a 30 dB stop-band attenuation is sufficient to provide good performance. We choose low-order IIR filters for computational saving.

Since there are group delays associated with the filters $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$, the correction term in coefficient adaptation is lagged. This will cause a delay in adaptation and hence reduce the convergence rate (Long et al., 1989, 1992). Fortunately, this reduction in convergence rate is counter balanced by the removal of the unwanted components in the adaptive filter input and error signals which accelerate convergence. We should choose filters with small group delays. In practice, very narrow transient bands are not necessary for the filters. Low-order IIR filters satisfy these requirements. The bandwidth of the filter is determined by the highest and the lowest oscillation frequencies of each hearing aid user.

2.4. Selection of the adaptive filter length

The goal of the feedback cancellation is not to match the adaptive filter response to the feedback paths at all frequencies, but only to match them at oscillation frequencies. Therefore, the adaptive filter length can be chosen to provide sufficient frequency resolution to match frequency responses at all oscillation frequencies. We chose the number of coefficients to be at least twice the number of oscillation frequencies so that enough degrees of freedom are provided. This is especially important because new oscillation frequencies may occur in the band of interest when a feedback canceller is used. Therefore, the adaptive filter length should have headroom for covering new oscillation frequencies.

3. Convergence analysis of band-limited adaptive feedback cancellation

For an adaptive feedback cancellation system, convergence behavior and stability are important and closely related to the oscillation suppression ability and hence the output sound quality. To offer better oscillation suppression, adaptive filters should have sufficient convergence rates so as to track the variation of dynamic feedback paths, which is caused by the time-varying acoustic feedback paths or the gain changes of the AGC. In this section, we will conduct a convergence analysis to show that the proposed band-limited adaptive feedback canceller can provide higher convergence rates than wideband feedback cancellers. The analysis is performed in the frequency domain so that the operations of band-limited adaptive filtering are easily illustrated. In addition, the phase requirement of the bandpass filters $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$ for stable adaptation of the band-limited adaptive filtered-X LMS filter will be derived. This phase condition is shown to be similar to the results found by Morgan (1980), Elliott et al. (1987) and Snyder and Hansen (1994). The following mathematical derivation starts from the band-limited frequency domain convergence analysis, which is a different approach from that in the previous works. The intermediate results of the derivation will be used for explaining the advantages of the band-limited approach.

In the analysis, the forward path of the closed-loop feedback system is disconnected, and the AGC and OCL are not taken into account. Standard LMS adaptation is used in the derivation for simplicity. It is possible to generalize to normalized LMS adaptation because the formulation is similar except that the step-size is normalized by the signal power. The coefficients of the LMS adaptive filter are updated by

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + \mu\varepsilon(n)\boldsymbol{x}_M(n), \tag{1}$$

where $\boldsymbol{x}_M(n) = (x(n), x(n-1), \ldots, x(n-M+1))^{\mathrm{T}}$. For the wideband feedback canceller case, $x(n) = u(n)$ and $\varepsilon(n) = e(n)$. For the band-limited feedback case, $x(n) = u_f(n)$ and $\varepsilon(n) = e_f(n)$.

Expanding the recursion in Eq. (1) L times, we obtain

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n-L+1) + \mu(\boldsymbol{x}_M(n), \boldsymbol{x}_M(n-1), \ldots, \boldsymbol{x}_M(n-L+1))\boldsymbol{\varepsilon}_L(n), \tag{2}$$

where $\varepsilon_L(n) = (\varepsilon(n), \varepsilon(n-1), K, \varepsilon(n-L+1))^{\mathrm{T}}$.

Let $x_{n-i,M}(m) = x(m)\alpha_M(m-(n-i))$, where $i = 0, 1, \ldots, L-1$, and $\alpha_M(m)$ is a rectangular window with the length M and is given by

$$\alpha_M(m) = \begin{cases} 1, & -M+1 \leqslant m \leqslant 0, \\ 0, & \text{otherwise.} \end{cases}$$

The Fourier transform of the sequence $\{x_{n-i,M}(m)\}$ is

$$X_{n-i,M}(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{m=-\infty}^{\infty} x_{n-i,M}(m)\mathrm{e}^{-\mathrm{j}\omega m} = \sum_{m=n-i-M+1}^{n-i} x(m)\mathrm{e}^{-\mathrm{j}\omega m}. \tag{3}$$

By taking the conjugate,

$$X^*_{n-i,M}(\mathrm{e}^{\mathrm{j}\omega}) = \mathrm{e}^{\mathrm{j}(n-i)\omega}(1, \mathrm{e}^{-\mathrm{j}\omega}, \ldots, \mathrm{e}^{-\mathrm{j}(M-1)\omega})\boldsymbol{x}_M(n-i). \tag{4}$$

Hence,

$$\boldsymbol{v}^{\mathrm{H}}(\mathrm{e}^{\mathrm{j}\omega})\boldsymbol{x}_M(n-i) = \mathrm{e}^{-\mathrm{j}(n-i)\omega}X^*_{n-i,M}(\mathrm{e}^{\mathrm{j}\omega}), \tag{5}$$

where $\boldsymbol{v}(\mathrm{e}^{\mathrm{j}\omega}) = (1, \mathrm{e}^{\mathrm{j}\omega}, \ldots, \mathrm{e}^{\mathrm{j}(M-1)\omega})^{\mathrm{T}}$ and the superscript 'H' is the Hermitian operation.

Now consider the frequency response of the adaptive filter as

$$W_n(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{k=0}^{M-1} w_k(n)\mathrm{e}^{-\mathrm{j}\omega k} = \boldsymbol{v}^{\mathrm{H}}(\mathrm{e}^{\mathrm{j}\omega})\boldsymbol{W}(n). \tag{6}$$

Left-multiplying both sides of Eq. (1) by $\boldsymbol{v}^{\mathrm{H}}(\mathrm{e}^{\mathrm{j}\omega})$ and using Eqs. (5) and (6), we obtain the recursion for the frequency responses of the coefficients as follows:

$$W_{n+1}(\mathrm{e}^{\mathrm{j}\omega}) = W_{n-L+1}(\mathrm{e}^{\mathrm{j}\omega}) + \mu\mathrm{e}^{-\mathrm{j}\omega n}\Big(X^*_{n,M}(\mathrm{e}^{\mathrm{j}\omega}), \mathrm{e}^{\mathrm{j}\omega}X^*_{n-1,M}(\mathrm{e}^{\mathrm{j}\omega}), \ldots, \mathrm{e}^{\mathrm{j}(L-1)\omega}X^*_{n-L+1,M}(\mathrm{e}^{\mathrm{j}\omega})\Big)\varepsilon_L(n). \tag{7}$$

It can be shown that the above recursion can be written as (Appendix A of Chapter 4, Chi, 1999)

$$W_{n+1}(\mathrm{e}^{\mathrm{j}\omega}) = W_{n-L+1}(\mathrm{e}^{\mathrm{j}\omega}) + \mu\Big[X^*_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})\varepsilon_{n,L}(\mathrm{e}^{\mathrm{j}\omega})\Big] * A^*_M(\mathrm{e}^{\mathrm{j}\omega}), \tag{8}$$

where "$*$" denotes periodic convolution. $A_M(\mathrm{e}^{\mathrm{j}\omega})$ is the Fourier transform of the rectangular window $\{\alpha_M(m)\}$ and is given as $\mathrm{e}^{\mathrm{j}\omega(M-1)/2}(\sin\pi\omega)/(\pi\omega)$, and $\varepsilon_{n,L}(\mathrm{e}^{\mathrm{j}\omega})$ is the Fourier transform of $\{\varepsilon(m)\alpha_L(m-n)\}$. It can be shown that $\varepsilon_{n,L}(\mathrm{e}^{\mathrm{j}\omega})$ can be approximated by $\tilde{\varepsilon}_{n,L}(\mathrm{e}^{\mathrm{j}\omega})$ (Appendix B of Chapter 4, Chi, 1999).

$$\begin{aligned}\varepsilon_{n,L}(\mathrm{e}^{\mathrm{j}\omega}) &= H_2(\mathrm{e}^{\mathrm{j}\omega})F(\mathrm{e}^{\mathrm{j}\omega})Q_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega}) + H_2(\mathrm{e}^{\mathrm{j}\omega})S_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega}) \\ &\quad - \mathrm{e}^{-\mathrm{j}\Delta_1\omega}H_1(\mathrm{e}^{\mathrm{j}\omega})H_2(\mathrm{e}^{\mathrm{j}\omega})Q_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})W_{n-L+1}(\mathrm{e}^{\mathrm{j}\omega}).\end{aligned} \tag{9}$$

In which $Q_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})$ and $S_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})$ are the Fourier transforms of the windowed sequences $\{q_{n,L+M}(n)\}$ and $\{s_{n,L+M}(n)\}$, respectively.

Substituting the error into Eq. (8) and taking the expected values on both sides, we obtain the approximations as follows:

$$
\begin{aligned}
&E[W_{n+1}(\mathrm{e}^{\mathrm{j}\omega})] \ * \ A_M^*(\mathrm{e}^{\mathrm{j}\omega}) \\
&\quad = \Big\{ \Big(1 - \mu H_2^*(\mathrm{e}^{\mathrm{j}\omega}) H_3(\mathrm{e}^{\mathrm{j}\omega}) |H_1(\mathrm{e}^{\mathrm{j}\omega})|^2 \Phi_{qq,n}(\mathrm{e}^{\mathrm{j}\omega})\Big) \\
&\qquad \times E[W_{n-L+1}(\mathrm{e}^{\mathrm{j}\omega})] + \mu \mathrm{e}^{\mathrm{j}\Delta_1 \omega} H_1^*(\mathrm{e}^{\mathrm{j}\omega}) H_2^*(\mathrm{e}^{\mathrm{j}\omega}) H_3(\mathrm{e}^{\mathrm{j}\omega}) \\
&\qquad \times \big[F(\mathrm{e}^{\mathrm{j}\omega})\Phi_{qq,n}(\mathrm{e}^{\mathrm{j}\omega}) + \Phi_{sq,n}(\mathrm{e}^{\mathrm{j}\omega})\big]\Big\} \ * \ A_M^*(\mathrm{e}^{\mathrm{j}\omega}).
\end{aligned} \tag{10}
$$

The "independence assumption" mentioned in the textbook of Haykin (1993) is used (independence of input vectors and desired signal vectors is assumed to make convergence analysis possible). The short-time spectrum of $q(n)$ with a window size $L+M$ is $\Phi_{qq,n}(\mathrm{e}^{\mathrm{j}\omega})$, $\Phi_{qq,n}(\mathrm{e}^{\mathrm{j}\omega}) = E[|Q_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})|^2]$. The short-time cross-spectrum between $s(n)$ and $q(n)$ with a window size $L+M$ is $\Phi_{sq,n}(\mathrm{e}^{\mathrm{j}\omega})$, $\Phi_{sq,n}(\mathrm{e}^{\mathrm{j}\omega}) = E[S_{n,L+M}(\mathrm{e}^{\mathrm{j}\omega})Q_{n,L+M}^*(\mathrm{e}^{\mathrm{j}\omega})]$.

3.1. Stability conditions

From Eq. (10), the condition that makes $E[W_n(\mathrm{e}^{\mathrm{j}\omega})]$ converge at all frequencies for large n is

$$
|1 - \mu H_2^*(\mathrm{e}^{\mathrm{j}\omega}) H_3(\mathrm{e}^{\mathrm{j}\omega}) |H_1(\mathrm{e}^{\mathrm{j}\omega})|^2 \Phi_{qq}(\mathrm{e}^{\mathrm{j}\omega})| < 1, \tag{11}
$$

where $\Phi_{qq}(\mathrm{e}^{\mathrm{j}\omega})$ is the steady-state value of the short-time spectrum $\Phi_{qq,n}(\mathrm{e}^{\mathrm{j}\omega})$.

Eq. (11) generates the stability condition as follows:

$$
\mu < \min_{-\pi \leqslant \omega < \pi} \left\{ \frac{2\mathrm{Re}\{\mathrm{e}^{\mathrm{j}(\theta_3(\omega)-\theta_2(\omega))}\}}{|H_1(\mathrm{e}^{\mathrm{j}\omega})|^2 |H_2(\mathrm{e}^{\mathrm{j}\omega})| \, | \, H_3(\mathrm{e}^{\mathrm{j}\omega})| \Phi_{qq}(\mathrm{e}^{\mathrm{j}\omega})} \right\} \tag{12}
$$

where $\theta_2(\omega)$ and $\theta_3(\omega)$ denote the phase responses of $H_2(\mathrm{e}^{\mathrm{j}\omega})$ and $H_3(\mathrm{e}^{\mathrm{j}\omega})$, respectively.

To stabilize the LMS adaptation, the step-size parameter has to satisfy the above condition.

3.2. Phase matching of $H_2(\mathrm{e}^{\mathrm{j}\omega})$ and $H_3(\mathrm{e}^{\mathrm{j}\omega})$ in the band-limited adaptive feedback canceller

From the stability condition, we know that the band-limited adaptive filter will be unstable if there exists any ω such that

$$
\mathrm{Re}\{\mathrm{e}^{\mathrm{j}(\theta_2(\omega)-\theta_3(\omega))}\} > 0. \tag{13}
$$

To make the band-limited adaptive filter stable, we have to choose $H_2(\mathrm{e}^{\mathrm{j}\omega})$ and $H_3(\mathrm{e}^{\mathrm{j}\omega})$ such that

$$
-\frac{\pi}{2} < \theta_2(\omega) - \theta_3(\omega) < \frac{\pi}{2}, \text{ for all } \omega. \tag{14}
$$

That is, the phase responses of $H_2(\mathrm{e}^{\mathrm{j}\omega})$ and $H_3(\mathrm{e}^{\mathrm{j}\omega})$ should be very close to each other at all frequencies. Otherwise, the allowable value for μ will be very small, and the convergence rate will be low. In our implementation, we use $H_2(\mathrm{e}^{\mathrm{j}\omega}) = H_3(\mathrm{e}^{\mathrm{j}\omega})$.

3.3. Fast convergence of the band-limited adaptive feedback canceller

When the step-size parameter μ is chosen to satisfy this stability condition, the adaptive feedback canceller will converge at every frequency. If the convergence factor, the left-hand side of Eq. (11), at a certain frequency is small, the adaptive feedback canceller will converge fast at that frequency. When the oscillation components have been built up and introduce spectral peaks in $\Phi_{qq}(\mathrm{e}^{\mathrm{j}\omega})$ at the oscillation frequencies, the convergence factors at these frequencies will be very small so that the convergence rates are much higher than at other frequencies.

When oscillation components are suppressed but not completely eliminated, the highest peak of $\Phi_{qq}(\mathrm{e}^{\mathrm{j}\omega})$ would be mainly caused by components from the external signal $s(n)$, which has most of its energy at low frequencies. When the wideband LMS algorithm is used, the maximum step-size parameter for stable adaptation is determined by the highest spectral peak caused by $s(n)$ rather than by oscillation spectral peaks. Therefore, the step-size is restricted by the low-frequency spectral peaks and would be small so that adaptation at the oscillation frequencies becomes relatively slow. With band-limited algorithms, the spectral peaks caused by $s(n)$ in the low frequency band are removed by the bandpass filters. The maximum step-size determined by Eq. (12) would be greater than the wideband method. Thus, the convergence rates at oscillation frequencies are significantly increased. The band-limited adaptive feedback cancellers would cancel residual oscillation components more clearly than wideband algorithms as the feedback paths are changing. This assertion is further verified by computer simulations.

4. Computer simulations

The performance of adaptive feedback cancellation algorithms is evaluated using computer simulations of a wideband NLMS approach and the approach proposed in this paper. In the simulations, digitized speech material, at 16 kHz sampling rate, were used as input signals and were calibrated to 80 dB SPL. The hearing aid comprises an AGC, a hearing aid filter, and an output compression limiter (OCL). The AGC is set with compression threshold (CT) = 85 dB SPL, 1.5 ms attack time and 150 ms release time. The input signal level and the AGC CT are set so as to have the occurrence of AGC gain change. Thus, the performance affected by the AGC gain change can be assessed. The hearing aid filter, denoted as 'PCC', is used and the magnitude response is shown in Fig. 5(a). The volume control is set to provide 30–40 dB high frequency average gain on a 2-cc coupler. At the OCL output, digital samples are converted to analog signal output. Dynamic feedback paths measured from a KEMAR ear with hand movements close by are used in the simulations.

In the band-limited scheme, three identical second-order highpass IIR filters are used with cut-off frequencies around 2.2 kHz, 30 dB stop-band attenuation, and 3 dB pass-band ripple. Fig. 6 shows the magnitude and phase responses of the second-order IIR filter for $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ which are used in our simulation. Based on the pre-calculations, the pass-band can cover all oscillation frequencies. The same 32-tap NLMS adaptive filter is used for the wideband and band-limited feedback cancellers. We choose the step-size and forgetting factor parameters to be 0.125 and 0.984, respectively, for all simulations. A 12-sample delay is inserted in front of the adaptive filter for the wideband scheme. For the band-limited scheme, a 4-sample delay is placed prior to the bandpass filter $H_1(e^{j\omega})$ since the IIR filter $H_1(e^{j\omega})$ introduces approximately an 8-sample delay in the transient bands.

Based on these parameter settings, computer simulations are conducted to compare the performance

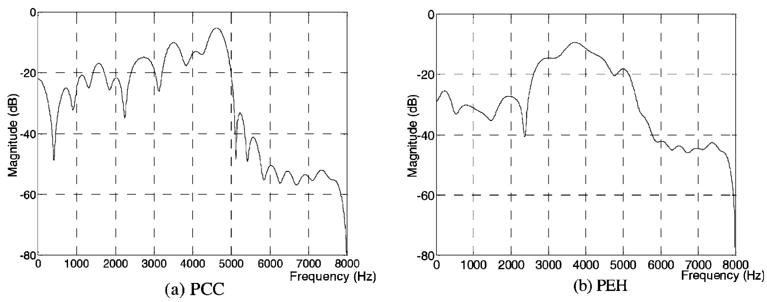

Fig. 5. Hearing aids filter responses used in computer simulation and subjective evaluation.

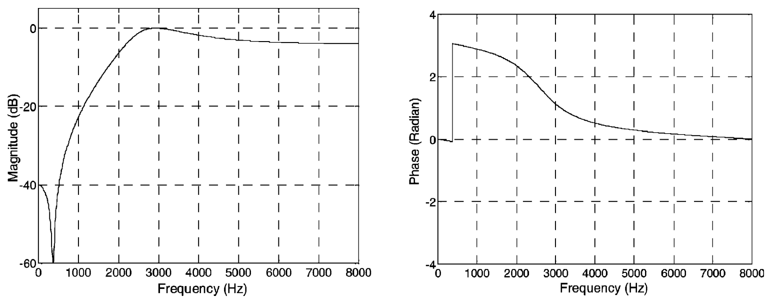

Fig. 6. Magnitude and phase responses of the second-order IIR filter for $H_1(e^{j\omega})$, $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$ which are used in our simulation.

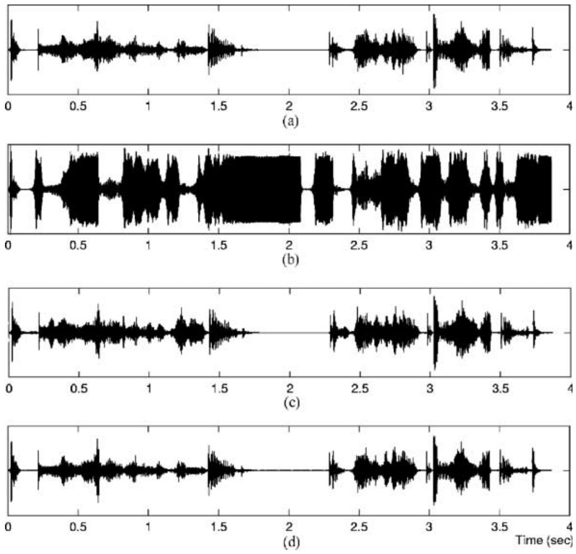

Fig. 7. Waveforms of the hearing aid outputs when the sentences "A boy falls from a window. The wife helps her husband." spoken by a male speaker is used as the input: (a) the reference signal (output of hearing aid without feedback), (b) without feedback cancellation, (c) with a wideband feedback canceller and (d) with a band-limited feedback canceller.

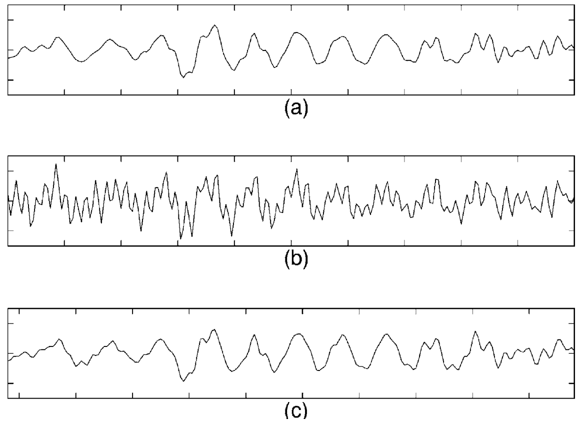

Fig. 8. A small interval (between 1.50 and 1.55 s) in the waveforms: (a) the reference signal, (b) with a wideband feedback canceller and (c) with a band-limited feedback canceller.

of different feedback cancellers. Fig. 7 shows waveforms of the hearing aid outputs when a male sentence is used as the input signal. Fig. 7(a) is the reference waveform, which is the hearing aid output without a feedback path and a canceller. Fig. 7(b) is the waveform of the hearing aid output without feedback cancellation. Fig. 7(c) and (d) show the waveforms of the hearing aid output when the wideband feedback canceller and the band-limited feedback canceller are used, respectively. Careful observation shows that the waveform resulting from the band-limited feedback canceller is a closer match to the reference waveform. Fig. 8 shows a small interval in the above waveforms and illustrates the advantage of the band-limited scheme since there are some residual oscillation components in the output waveform of the wideband scheme. By listening to the output sound, we clearly hear a metallic ringing sound that results from these residual components.

To observe the unwanted signal components which are introduced by feedback cancellers, we calculate the difference signals between the cancellation output $e(n)$ and the original input $s(n)$. Since the AGC will modulate the input signal, we apply AGC gain to $s(n)$ so as to calculate the difference signals. The averaged PSD functions of the difference signals are computed and shown in Fig. 9. As shown, the wideband feedback canceller introduces more unwanted signal components in the low frequency region than the band-limited scheme. In addition, there are spectral peaks in PSD of the wideband scheme which are unsuppressed residual oscillation components.

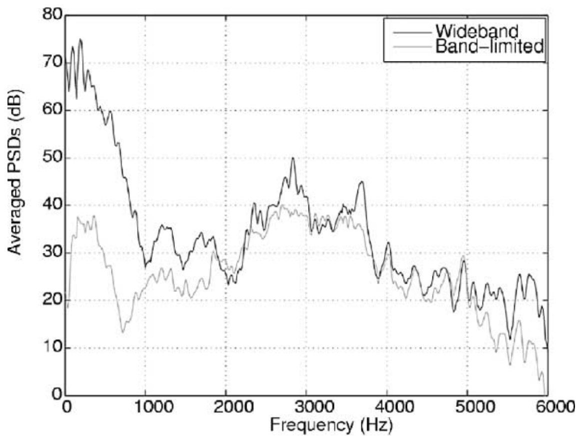

Fig. 9. Averaged PSDs of difference signals between $e(n)$ and AGC-modulated $s(n)$ when a male sentence is used as the input.

Another way to compare the performance of feedback cancellers is to see how much non-linear distortion is introduced at the hearing aid output. As described in Section 1, the variation caused by the adaptation of feedback cancellers will result in non-linear distortion. Here, we use the coherence between the reference signal and the hearing aid output as a metric of non-linear distortion (Dyrlund, 1989). Fig. 10 shows coherence functions calculated from the hearing aid output signals for the speech sentence. In the figure, the coherence of the wideband feedback cancellation algorithm is lower (more low frequency distortion) than that of the band-limited algorithm and has several deep notches, which are mainly caused by strong residual oscillation components. Note that the coherence function is not an adequate distortion measure for suboscillatory conditions.

In the following simulation, we observe the feedback suppression abilities and the convergence behavior of feedback cancellers. In the simulation, we use a speech-shaped noise at 60-dB SPL as input, shown in Fig. 11. To assess the feedback suppression efficiency, we calculate the misadjustment of the adaptive feedback cancellers within the band of interest which is defined in the band-limited adaptive feedback canceller. The misadjustment is calculated by using a normalized band-limited deviation (NBLD) and a maximum band-limited deviation (MBLD), which are defined as, at the nth iteration,

$$\mathrm{NBLD}(n) = \sqrt{\frac{\int_{\omega\in B} |H_n(e^{j\omega}) - \eta(n)F(e^{j\omega})|^2}{\int_{\omega\in B} |\eta(n)F(e^{j\omega})|^2}}, \quad (15)$$

$$\mathrm{MBLD}(n) = \max_{\omega\in B}\{|H_n(e^{j\omega}) - \eta(n)F(^{j\omega})|\}. \quad (16)$$

$H_n(e^{j\omega})$ is the instantaneous overall frequency response along the adaptive feedback cancellation path, which may include a delay or bandpass filters. $F(e^{j\omega})$ is the feedback path, $\eta(n)$ is the AGC gain signal, and B is the frequency band specified in the band-limited feedback canceller. Low NBLD and MBLD indicate a small feedback path gain (combination of the acoustic feedback path and the feedback canceller path). In the simulation, a fixed time-invariant feedback path and a hearing aid filter, with the open-loop transfer function shown in Fig. 3, are used. The aforementioned hearing-aid parameter-settings are used so that there is a 15 dB additional gain to the marginally stable gain without feedback cancellation. The band of interest B is $\{\omega: \omega \geqslant 2.2\ \text{kHz}\}$. For the wideband feedback canceller, $H_n(e^{j\omega}) = W_n(e^{j\omega})e^{-j\Delta\omega}$. For the band-limited feedback canceller, $H_n(e^{j\omega}) = W_n(e^{j\omega})H_1 \times (e^{j\omega})e^{-j\Delta_1\omega}$. $W_n(e^{j\omega})$ is the instantaneous frequency response of the adaptive filter.

All simulations start with zero adaptive filter coefficients. Fig. 12 shows the NBLD and MBLD learning curves for the wideband and band-limited feedback cancellers. The feedback cancellers start

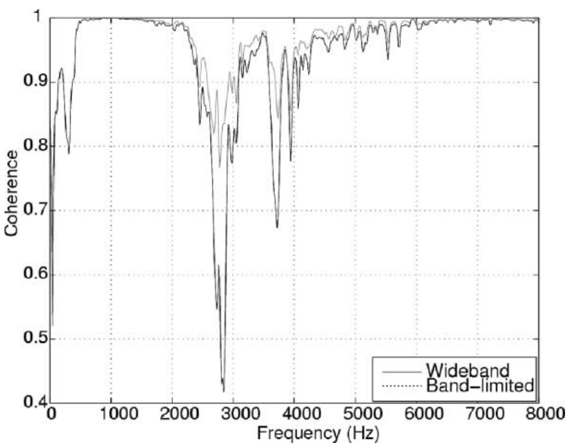

Fig. 10. Coherence functions between the reference signal and the hearing aid outputs with two feedback cancellers when a male sentence is used as the input.

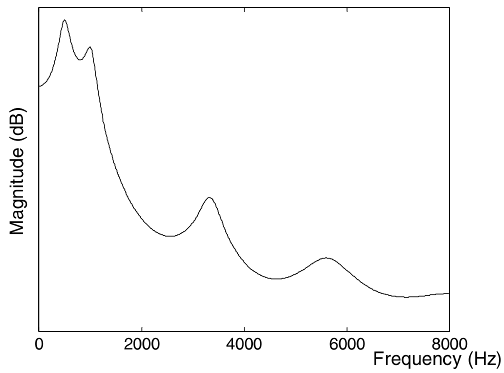

Fig. 11. Speech-shaped noises are used in computer simulation.

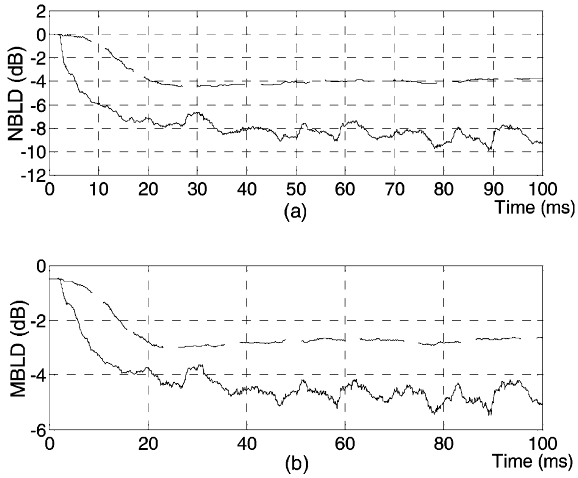

Fig. 12. NBLD and MBLD learning curves of two adaptive feedback cancellers (wideband: dashed line; band-limited: solid line) for speech shaped noise inputs.

with fast adaptation and then slow down the adaptation to steady state values. In the figure, the band-limited scheme takes a shorter time to reach certain values of NBLD and MBLD than the wideband scheme. This verifies that the band-limited canceller could suppress feedback faster than the wideband canceller. Moreover, at steady states, the NBLD and MBLD of the band-limited method are several dB below that of the wideband method. This can explain why the residual oscillation components which exist in the output waveform of the wideband scheme are not found in the band-limited scheme.

4.1. Further examination of AGC effects on adaptation

Adaptive filters are used to estimate the feedback path which is composed of the microphone, the acoustic feedback path, the receiver, and the AGC. In addition to the dynamic acoustic feedback path, the modulation caused by the AGC gain change also makes the feedback path time varying. During the attack and release periods, the AGC gain is not fixed but changes at each sample. Therefore, the adaptive filter does not only track the variation caused by the dynamic acoustic feedback path but also adapts the magnitude of the coefficients to track the gain variation of the AGC. If the tracking speed of the adaptive filter is not comparable with the attack and release rates of the AGC, brief oscillation will occur during the attack and release periods. With a speech sentence input, we can hear a brief metallic sound at the onset of a burst in the output speech signal if the adaptation speed of the feedback canceller is inadequate. One way to alleviate this problem is to apply the same amplification as the AGC gain to the output signals of the adaptive filter. But, this requires access to the digital representation of the AGC gain control signal.

To examine the effects of AGC gain modulation on the misadjustment of adaptive feedback cancellers, we conducted another computer simulation. We synthesized a 56-dB SPL speech-shaped noise with a 90-dB short burst in the middle and used it as the input signal, as shown in Fig. 13(a). The burst will trigger an AGC attack at the onset, and the AGC gain control signal is as shown in Fig. 13(b). Figs. 13(c) and (d) show NBLD and MBLD curves of the wideband and the band-limited feedback cancellers, respectively. When the AGC gain drops, the misadjustments of feedback cancellers at the oscillation frequencies increase abruptly and then stay at high levels for a period of time. During the period of high misadjustment, both feedback cancellers fail to track the feedback paths at oscillation frequencies. Fortunately, this does not cause a problem since the AGC gain is low during this period so that the second Nyquist criterion (magnitude condition) is not satisfied. Therefore, there is no oscillation problem in this period. Thereafter, the adaptive filters adapt to reduce the magnitude of coefficients in order to track the AGC gain change. After the AGC gain recovers to a certain level, if the adaptive feedback cancellers have not adapted the coefficients to follow the current levels of the feedback paths closely enough, oscillation may happen. Therefore, the adaptation of the feedback cancellers should be fast enough to prevent this problem. From Figs. 13(c) and (d), we observe that the band-limited adaptive feedback canceller has fast adaptation and is able to reduce the misadjustment quickly while the AGC gain is recovering.

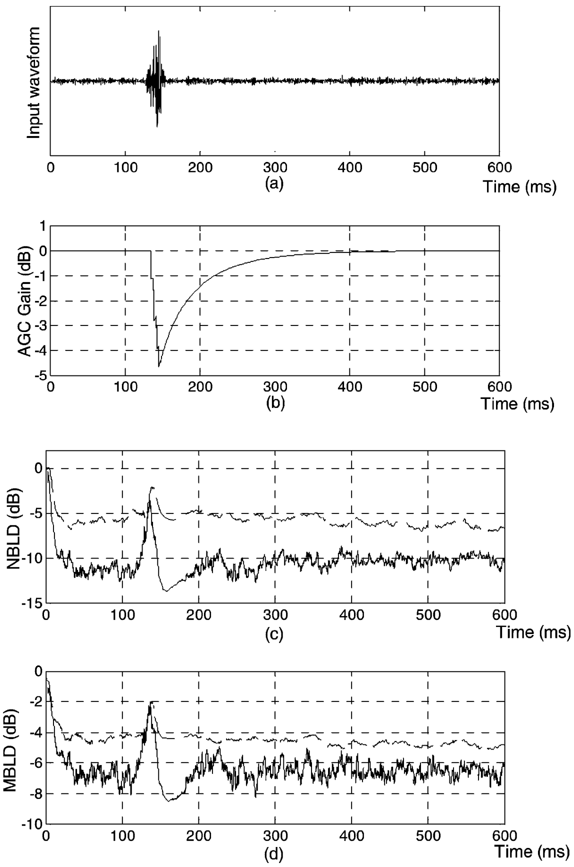

Fig. 13. (a) Input signal (speech shaped noise with a burst), (b) the AGC gain signal, (c) NBLD curves of two adaptive feedback cancellers (wideband: dashed line; band-limited: solid line) and (d) MBLD curves of two adaptive feedback cancellers (wideband: dashed line; band-limited: solid line).

5. Subjective evaluation

We conducted the listening test using normal hearing subjects to compare the performance of the wideband and the proposed band-limited feedback cancellation algorithms. Feedback cancellation algorithms have been implemented on a portable digital hearing aid prototype, which was designed by the Hearing Aid Research Department of the House Ear Institute and uses a Motorola DSP56001 processor.

The subjective evaluation was conducted by letting human subjects listen and grade the sound quality of the hearing aid output sound files which were recorded from a KEMAR ear canal. The sound material included three sentences spoken by a male talker and three sentences spoken by a female talker. We used a modified A–B comparison approach as the evaluation method. In addition to the A–B comparison, we asked the subjects to grade how much better the preferred sentence was.

Six subjects with normal audiometric thresholds ($<$ 25 dB HL) participated in the subjective evaluations. Each subject listened to three repetitions of two sound files, picked the one with the better sound quality, and also gave a preference score on a 0–5 scale (0 = 'no difference', 1 = 'almost negligibly better', 5 = 'much better'). To reasonably interpret the results, we use 'signed' preference scores to represent how much better the band-limited method is. Therefore, a positive score represents the situation where the band-limited method is better than the wideband method, and a negative score means that the wideband method provides better sound quality than the band-limited method.

Tables 1 and 2 show the averaged signed preference scores of six subjects with different hearing aid volumes control settings, hearing aid filters, input (male and female sentences), and input sound pressure levels (65 and 80 dBA). Two volume control settings are 'marginally stable gain' [1] (denoted as MSG) and 'at 10 dB above the marginally stable gain' (denoted as MSG + 10). PCC and PEH are two samples of hearing aid filters. Their magnitude responses are shown in Fig. 5.

Table 1
The signed preference scores with the hearing aid filter PCC (on a 0–5 scale, positive: band-limited is better, negative: wideband is better)

SPL	VC	Male sentence	Female sentence
65 dBA	MSG	0.167	2
	MSG + 10	0.833	3.333
80 dBA	MSG	1.167	2.5
	MSG + 10	1.5	2.333

VC refers to volume control. MSG refers to marginally stable gain.

[1] The marginally stable gain is defined as the gain at which the hearing aid is almost in the oscillatory state.

Table 2
The signed preference scores with the hearing aid filter PEH (on a 0–5 scale, positive: band-limited is better, negative: wideband is better)

SPL	VC	Male sentence	Female sentence
65 dBA	MSG	0.167	0.333
	MSG + 10	0.333	0.667
80 dBA	MSG	0.167	1
	MSG + 10	0.667	1.167

VC refers to volume control. MSG refers to marginally stable gain.

PEH has larger low-frequency attenuation than PCC.

We observe that the distribution of the scores varies with hearing aid filters, stimuli, and volume control settings. Since PEH has large low-frequency attenuation, differences between these two algorithms are not significant for speech signals. In general, the sound quality that the band-limited feedback canceller provides is better than the wideband method when speech material is used as stimuli.

6. Summary and conclusion

In this paper, we address the problems of AFC and investigate how distortion is introduced by the coefficient variation in the adaptive feedback canceller. We propose a new approach of feedback cancellation algorithm which has better performance than the wideband LMS approach with speech signal inputs. By concentrating the adaptive filtering on the bands which are known to have oscillation frequencies and by using normalized filtered-X LMS techniques, the distortion is reduced and the feedback cancellation efficiency is increased. This yields a noticeable improvement in output speech quality.

Acknowledgements

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