$$U'(n) = \begin{cases} (1/OT)\cdot(2\cdot n - 3\cdot n^2\ /OT), & n \le OT \\ 0, & n \ge OT+1 \end{cases}$$

$$U(n) = n^2 - n^3\ /OT$$

Here:
n = sample number (discrete time)
U'(n) = derivative of flow
U(n) = flow
OT = "open time" = INT{OPENQ * FSR / FSRC}
where
OPENQ = "open quotient" = pulse width/source period
FSR = sample rate
FSRC = fundamental frequency

Figure 2.11. Equations describing the KGLOTT88 glottal pulse model. [22]. The glottal flow derivative pulse is modeled using an inverted parabola with a linear term added to tilt and lower the end of the pulse

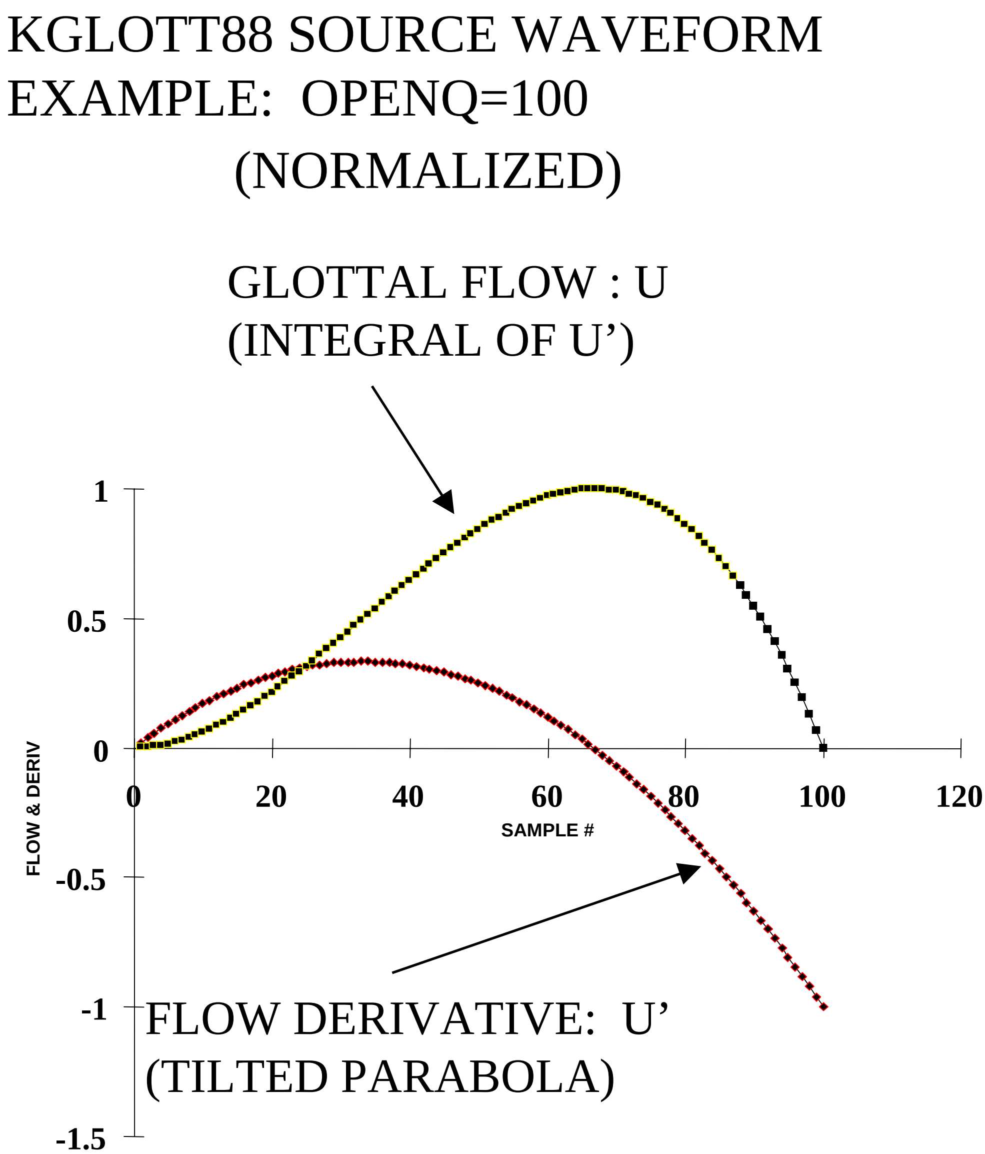
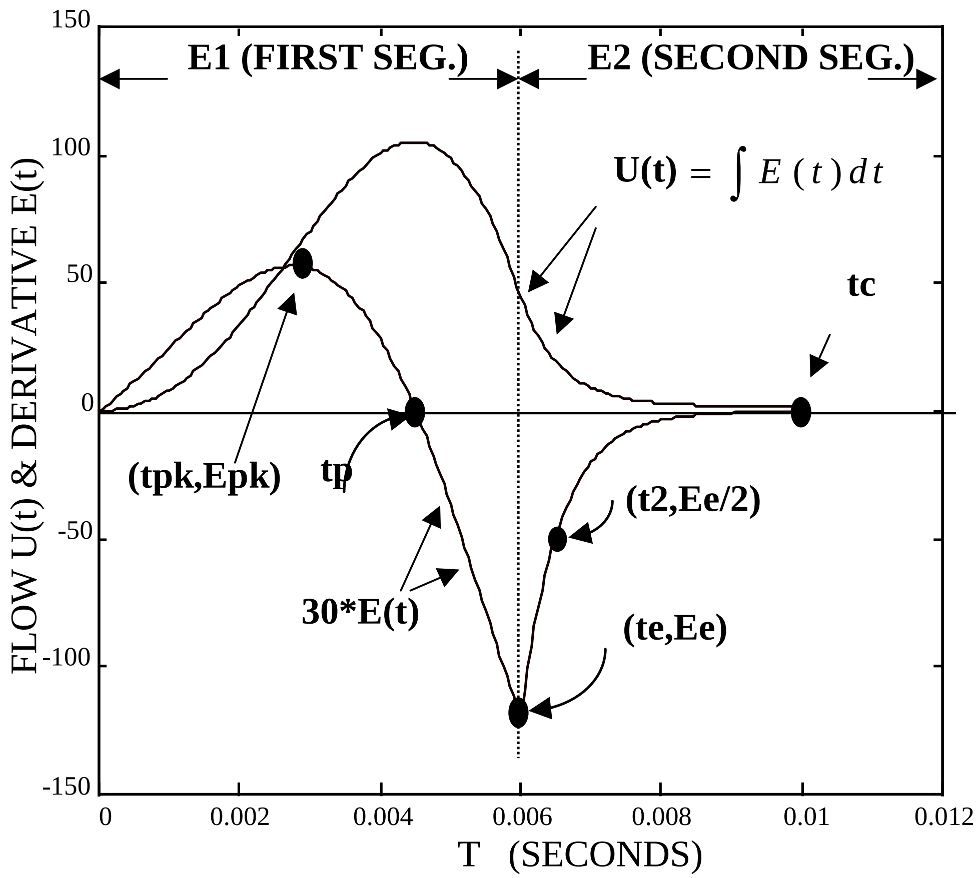


Figure 2.12. Plot of an example KGLOTT88 glottal flow derivative pulse U' and its integral U.



$$E(t)=\begin{cases} E_1(t)=E_0\, e^{\alpha t}\sin\omega_g t, & t\le t_e \\ E_2(t)=-E_e\, e^{-\varepsilon(t-t_e)}, & t_e \lhd t\le t_c \\ E_{2B}(t)=-E_e\, e^{-\varepsilon(t-t_e)}+m(t-te), & t_e \lhd t\le t_c \end{cases}$$

Figure 2.13. Simplified LF model used to fit the calculated flow derivative [31]. Both glottal flow (U) and its derivative (E) are shown. Four parameters (*tp*, *te*, *Ee*, *t2*) are major features that can define this model. The maxima defines tpk and Epk, which are additional quantities useful in generation of the LF parameters (Figs 2.15 - 2.16). A fifth parameter *m* is optionally added to supply a linear term to the second segment for improved fit, resulting in the second version E_{2B} of the second segment. Parameters ε and *Eo* are found by solving a set of simultaneous nonlinear equations.

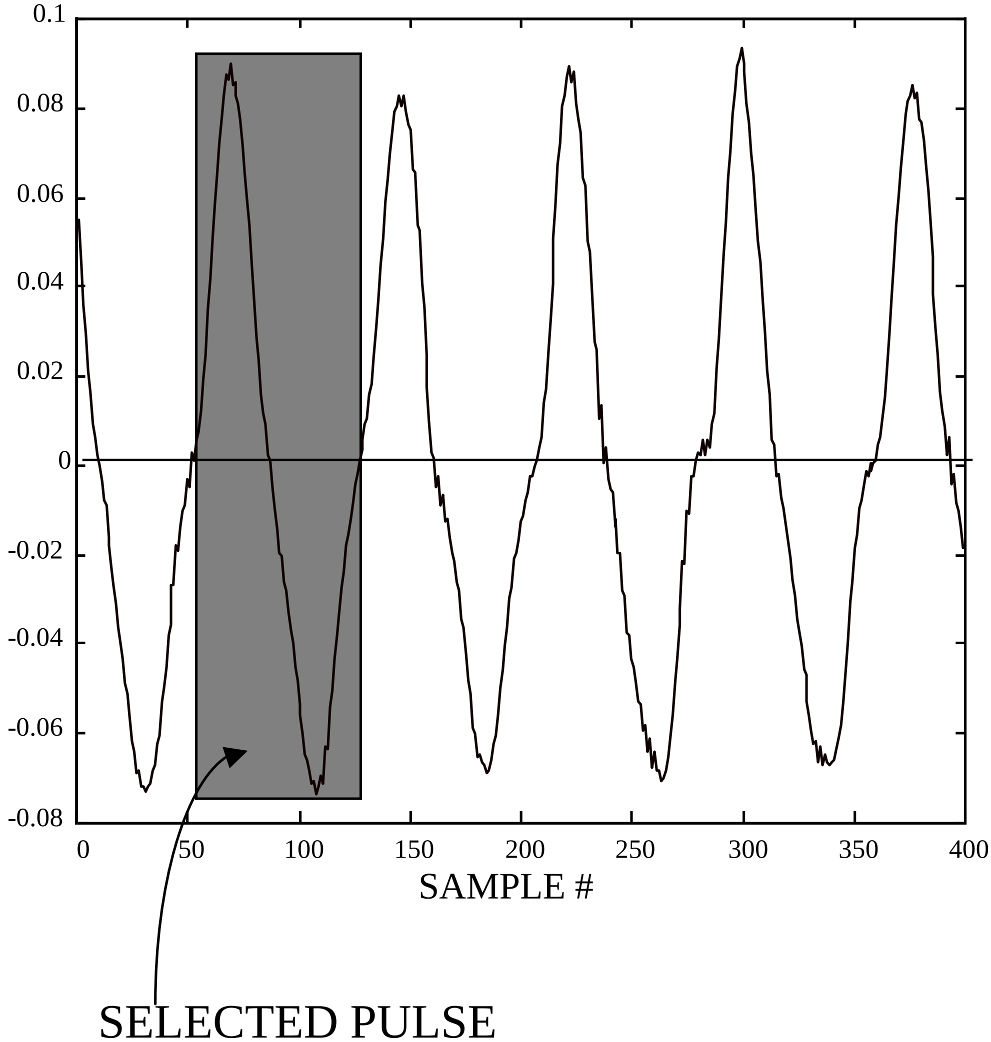


Figure 2.14. Selection of a single pulse from the glottal flow derivative time series. The LF model is least squares fitted to the selected pulse.

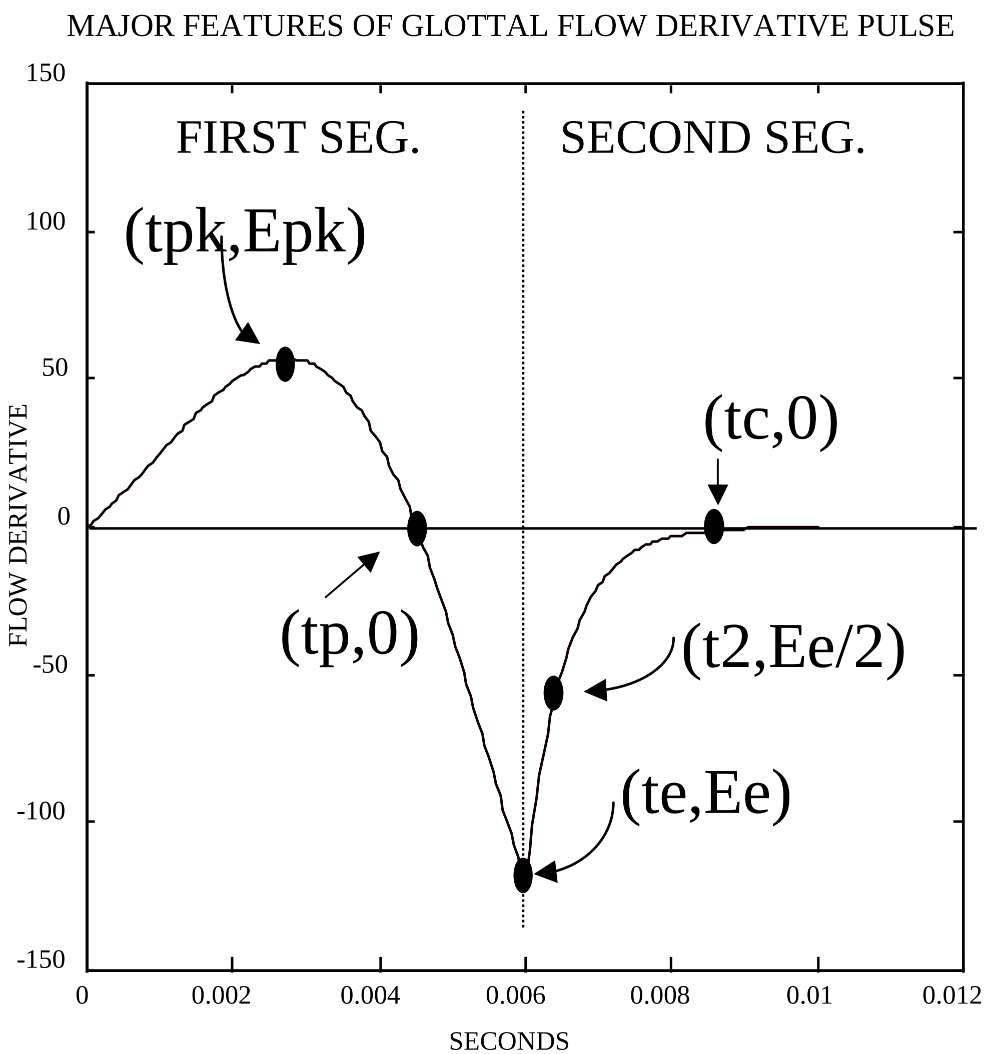


Figure 2.15. Identification of automatically acquired major features of the raw inverse filtered flow derivative. Clearly identifiable points on the pulse provide values for: Epk = positive peak value, tpk =– time of positive peak, Ee = negative peak value, te-= time of negative peak, Ee2 = half value of the second segment (return phase), and t2 = time of Ee2. The starting time is implicitly taken as zero at the start of the pulse data, and tc is set by the end of the pulse data.

1. $$\omega_g = \frac{\pi}{t_p}$$

2. $$\alpha = \frac{1}{\left(t_e - t_{pk}\right)} \cdot \ln\left(\frac{-E_e}{E_{pk}} \cdot \frac{\sin\left(\omega_g t_{pk}\right)}{\sin\left(\omega_g t_e\right)} \right)$$

3. $$t_{pk} = \frac{1}{\omega_g} \cdot \tan^{-1}\left(\frac{-\omega_g}{\alpha} \right), \qquad \alpha \lhd 0$$

$$t_{pk} = \frac{1}{\omega_g} \cdot \tan^{-1}\left(\pi + \frac{-\omega_g}{\alpha} \right), \qquad \alpha \geq 0$$

4. $$E_0 = \frac{-E_e}{e^{\alpha t_e} \sin\left(\omega_g t_e\right)}$$

5. $$\varepsilon = \frac{\ln(2)}{\left(t_2 - t_e\right)}$$

Figure 2.16. Equations used to define the first approximation of the LF fit by use of the major features (Fig. 2.15) automatically acquired from the raw inverse filtered glottal flow derivative pulse. The major features give rise to six values: Epk, tpk, Ee, te, tp, and t2, which are then used to determine five approximate values of LF parameter values by evaluating equations 1 – 5 in sequence. Since this system of equations is over determined (i.e., there are more feature parameters than needed), it was found useful to use equation 2, which combines two features as the ratio of Ee/Epk, a physically significant quantity.

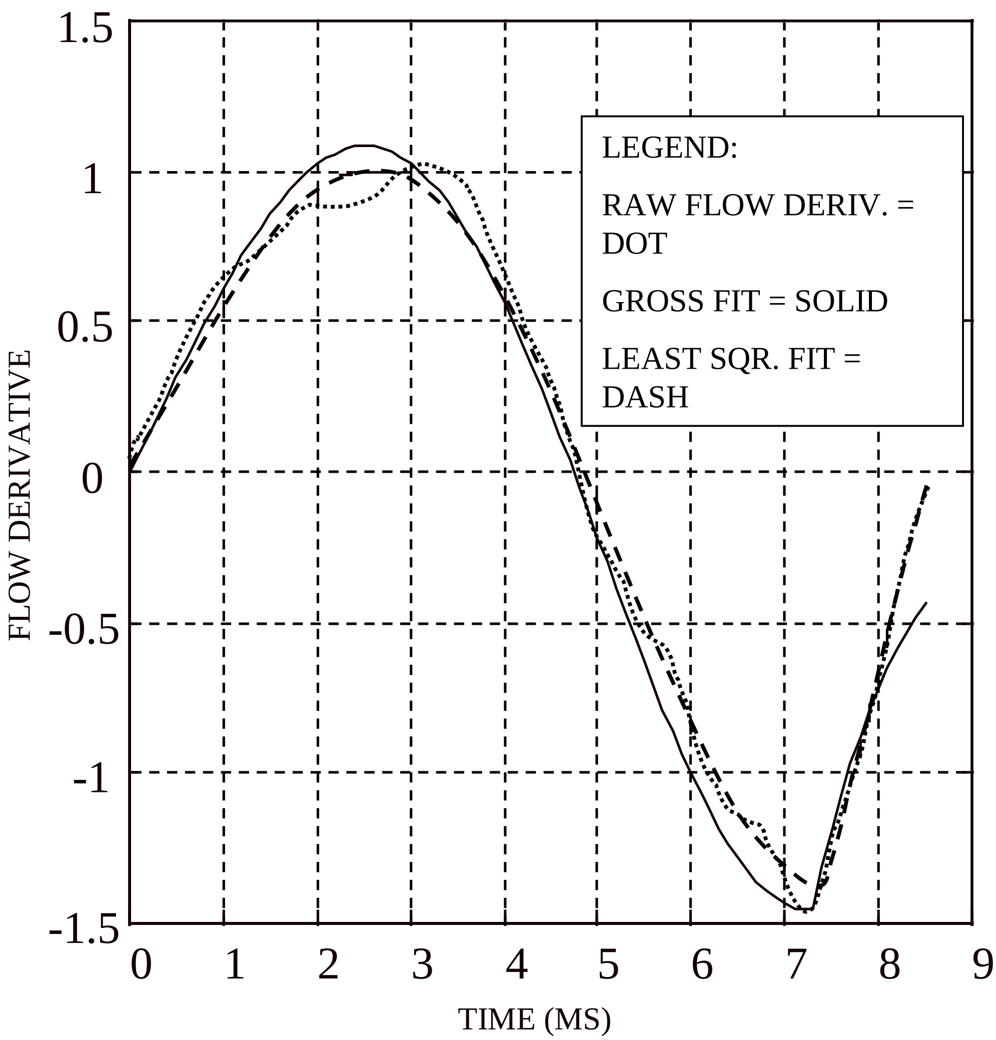


Figure 2.17. Example of fitting the LF model to a selected pulse. The dots represent the raw pulse, the line shows the first approximation used as a starting point for the least squares minimization, and the dash shows the final least squares fit.

1 - 3 solved simultaneously:

1. $$\alpha = \frac{1}{(t_e)} \cdot \ln\left(\frac{-E_e}{E_{pk}} \cdot \sin(\omega_g t_e) \right)$$

2. $$t_{pk} = \frac{1}{\omega_g} \cdot \tan^{-1}\left(\frac{-\omega_g}{\alpha} \right), \qquad \alpha \triangleleft 0$$

$$t_{pk} = \frac{1}{\omega_g} \cdot \tan^{-1}\left(\pi + \frac{-\omega_g}{\alpha} \right), \qquad \alpha \geq 0$$

3. $$E_0 = \frac{1}{e^{\alpha t_{pk}} \sin(\omega_g t_{pk})}$$

4. $$\varepsilon = \frac{\ln(2)}{(t_2 - t_e)}$$

5. $$m = \frac{-E_e}{t_c} \cdot e^{-\varepsilon t_c}$$ (optional parameter to force final value to zero)

Figure 2.18. Equations used to fit the raw pulse to obtain an optimized set of LF parameters in the least squares sense. The least squares fit is carried out using major feature parameters tp, Ee, te, and t2 as four independent degrees of freedom. Equations 1 - 3 form a simultaneous nonlinear set that are then solved to obtain the LF parameters using the simple sequential solution algorithm. Equation 5 is a parameter used in the optimization with the optional constraint to fix the final value of the pulse to zero.