# STEADY-STATE ANALYSIS OF CONTINUOUS ADAPTATION SYSTEMS FOR HEARING AIDS WITH A DELAYED CANCELLATION PATH

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**ABSTRACT**

*Acoustic feedback is a phenomenon that is both annoying to hearing-aid users and reduces the maximum usable gain of hearing-aid devices. To avoid acoustic feedback, continuous adaptation systems have been proposed in the past. This paper studies analytically, for the first time, the steady-state convergence behavior of adaptive algorithms when operating in continuous adaptation to reduce acoustic feedback when delays are used in the cancellation path. It is shown that an improvement in the adaptive filter estimate is possible depending on the type of feedback path and on the number of delays used in the cancellation path.*

**Keywords***: hearing aids, acoustic feedback, adaptive filters.*

## 1. INTRODUCTION

A major complaint of hearing-aid users is acoustic feedback which is perceived as whistling or howling (at oscillation) or distortion (at sub-oscillatory intervals). This feedback occurs, typically at high gains, because of leakage from the receiver to the microphone (Fig. 1). Acoustic feedback suppression in hearing-aids is important since it can increase the maximum insertion gain of the aid. The acoustic path transfer function can vary significantly depending on the acoustic environment [1]. Hence, effective acoustic feedback cancellers must be adaptive.

This paper analyzes the performance of adaptive filtering algorithms [2] when correlated inputs are used in continuous adaptation systems for hearing aids. It will be shown that any algorithm that attempts to approximate the Wiener solution in such systems will obtain a biased estimate of the feedback path due to the cross-correlation between the input and output signals of the hearing aid. Although previous works have addressed the necessity of decorrelation mechanisms to better estimate the feedback path, and a delayed cancellation path has been proposed [3], [4], none of these works has shown an analysis for the bias. This paper proposes analytical expressions to predict the bias; properties of continuous adaptation systems based on these analytical expressions are then discussed.

## 2. STEADY-STATE ANALYSIS USING CONTINUOUS ADAPTATION

A Wiener filter estimates a system transfer function $H(z)$ given the input signal to the system $s(n)$ and a corrupted system output signal $d(n)$. The corrupted signal is assumed to be the convolution

[1]This work was supported in part by NIH, and by a grant from CNPq/National Research Council in Brazil.

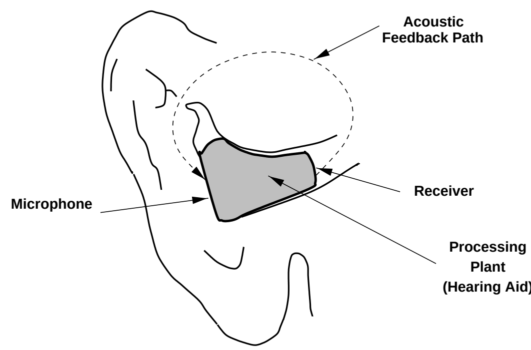


Figure 1: Acoustic feedback in a hearing aid coupled with a human ear.

between the system impulse response and the input signal $[s(n) * h(n)]$ added to a noisy component $r(n)$. It can be written as

$$d(n) = s(n) * h(n) + r(n). \tag{1}$$

In most cases, $r(n)$ is assumed to be white noise. The Wiener filter estimate [5] for $H(z)$ is a vector $\hat{\mathbf{h}}$ with the impulse response coefficients given by

$$\hat{\mathbf{h}} = \mathbf{R}^{-1}\mathbf{p}_{sd} \tag{2}$$

where

$$\mathbf{R} \triangleq E\{\mathbf{s}(n)\mathbf{s}^T(n)\} \tag{3}$$

$$\mathbf{p}_{sd} \triangleq E\{\mathbf{s}(n)d(n)\} \tag{4}$$

$$\mathbf{s}(n) \triangleq [s(n)\, s(n-1)\, \cdots\, s(n-N)]^T \tag{5}$$

and $N$ is the order of the Wiener estimate.

Now consider the case where the interference signal $r(n)$ is not white and is derived from the input signal $s(n)$ by a linear system. It is easy to prove that (2) will not lead to a proper estimate of the impulse response of $H(z)$. Suppose that $r(n)$ is derived from

$$r(n) = s(n) * q(n) \tag{6}$$

where $q(n)$ is the impulse response of an unknown system that derives the noisy signal $r(n)$ from the input signal $s(n)$. It is possible to show that the Wiener estimate impulse response vector $\hat{\mathbf{h}}$ deviates from the impulse response of $H(z)$ by

$$\boxed{\boldsymbol{\varepsilon} = \mathbf{R}^{-1}\mathbf{p}_{sr}} \tag{7}$$

where $\boldsymbol{\varepsilon}$, the bias, is defined as

$$\boldsymbol{\varepsilon} = \hat{\mathbf{h}} - \mathbf{h} \tag{8}$$

and $\mathbf{h}$ is a vector with the impulse response of $H(z)$. It should be noted that the entries of the vector $\boldsymbol{\varepsilon}$ contain a Wiener estimate for the impulse response $q(n)$.

### 2.1. Application to a feedback reduction system

In this section, expressions for the bias are derived for the case of a delay in the cancellation path. The superscript c will be used to indicate a delay in the cancellation path. All the vectors are assumed to have the Wiener Filter length ($N+1$ taps - $N^{th}$-order estimate) except where indicated with subscripts.

In the analysis, the bias in the feedback reduction system will be defined as

$$\boldsymbol{\varepsilon} = \mathbf{w} - \mathbf{f} \tag{9}$$

where $\mathbf{w}$ is a vector with the Wiener estimate for a hearing aid plant based on the input $[s(n)]$ and reference $[d(n)]$ signals to the adaptive filter [5] and $\mathbf{f}$ is a vector containing the feedback path impulse response.

### 2.2. The bias with a delayed cancellation path

In the case of the hearing aid plant shown in Fig. 2, the Wiener filter will be able to estimate the last $N+1$ terms of the impulse response, since the first $\Delta$ samples of the impulse response will be approximated by zeroes.

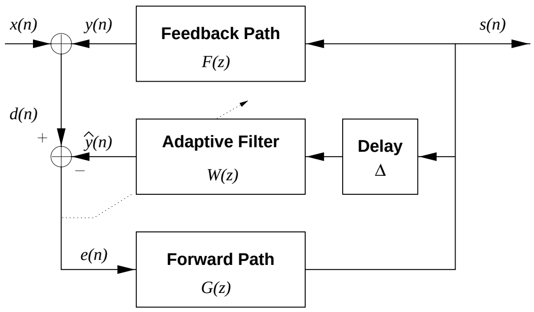


Figure 2: Model of a hearing aid plant with a delayed cancellation path.

It is also assumed that $F(z)$ can be divided into two components

$$F_1(z) = \mathbf{f}_{1,\Delta}^T \mathbf{T}_\Delta(z^{-1}) \tag{10}$$

$$F_2(z) = \mathbf{f}_2^T \mathbf{T}(z^{-1}) \tag{11}$$

where $\mathbf{f}_{1,\Delta}$ is a vector with the first $\Delta$ samples of the impulse response, $\mathbf{f}_2$ is a vector with the remaining $N+1$ samples, and $\mathbf{T}_\Delta(z)$ and $\mathbf{T}(z)$ are defined as

$$\mathbf{T}_\Delta(z) \triangleq [1\ z\ \cdots\ z^\Delta]^T \tag{12}$$

$$\mathbf{T}(z) \triangleq [1\ z\ \cdots\ z^N]^T. \tag{13}$$

The overall input-output relation for Figs. 2 and 3 are identical. Consequently, the reference signal can thus be written as

$$d(n) = y_1(n) + y_2(n) + x(n) \tag{14}$$

where

$$y_1(n) \triangleq \mathbf{f}_{1,\Delta}^T \mathbf{s}_\Delta(n) \tag{15}$$

$$y_2(n) \triangleq \mathbf{f}_2^T \mathbf{s}(n). \tag{16}$$

The components $y_1(n)$ and $y_2(n)$ are illustrated in Fig. 3.

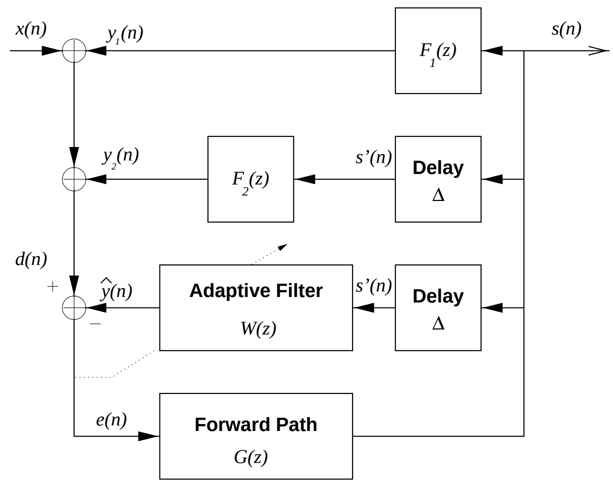


Figure 3: Model of a hearing aid plant with a delay in the cancellation path and with the feedback path divided into two components.

It is clear that the bias will contain two components in this case; one, due to the transfer function between $x(n)$ and $s(n)$, and a second bias term due to the output of $F_1(z)$. The bias can then be written as

$$\boldsymbol{\varepsilon}_2^c = \mathbf{R}^{-1}\mathbf{p}_1^c + \mathbf{R}^{-1}\mathbf{p}_2^c \tag{17}$$

where

$$\mathbf{p}_1^c \triangleq E\{\mathbf{s}(n-\Delta)x(n)\} = E\{\mathbf{s}(n)x(n+\Delta)\} \tag{18}$$

$$\mathbf{p}_2^c \triangleq E\{\mathbf{s}(n-\Delta)y_1(n)\} = E\{\mathbf{s}(n)\mathbf{s}_\Delta^T(n+\Delta)\}\mathbf{f}_{1,\Delta} \tag{19}$$

There is, however, another term in the bias vector due to approximating the first $\Delta$ samples of $F(z)$ with zeroes. This term can be written as

$$\boldsymbol{\varepsilon}_{1,\Delta}^c = \mathbf{0} - \mathbf{f}_{1,\Delta} = -\mathbf{f}_{1,\Delta}. \tag{20}$$

Thus, the total bias should be equal to

$$\boldsymbol{\varepsilon}^c = \begin{bmatrix} \boldsymbol{\varepsilon}_{1,\Delta}^c \\ \boldsymbol{\varepsilon}_2^c \end{bmatrix} = \begin{bmatrix} -\mathbf{f}_{1,\Delta} \\ \mathbf{R}^{-1}\mathbf{p}_1^c + \mathbf{R}^{-1}\mathbf{p}_2^c \end{bmatrix}. \tag{21}$$

## 3. THE USE OF DELAYS IN THE CANCELLATION PATH FOR DECORRELATING THE HEARING-AID INPUT AND OUTPUT SIGNALS

Determining quantitative relations for the bias as a function of the input signal statistics, the delay ($\Delta$) in the cancellation path and the forward path gain ($G_c$) is extremely complicated for arbitrary signals. This section analyzes the case where the input is an $M^{th}$-order auto-regressive sequence ($M$ poles $\rho_i, i = 1, \cdots, M$) to provide insight into the effect of the delays on the bias. The forward path is modeled as $G(z) = G_c z^{-1}$. Simulations with speech-shaped noise follow.

### 3.1. Bias reduction by use of delays in the cancellation path

This section presents analytical expressions for the cross-correlation vector $\mathbf{p}_1$ and the autocorrelation matrix $\mathbf{R}$ in the case that a delay is introduced in the cancellation path (in the input to the adaptive filter, as shown in Fig. 2).

#### 3.1.1. *Calculation of* $\mathbf{p}_1^c$

From Fig. 2, it is clear that the reference signal $d(n)$ can be written as

$$d(n) = x(n) + \mathbf{s}_{N+\Delta+1}^T \mathbf{f}_{N+\Delta+1}. \tag{22}$$

The error signal and the output signal are related by

$$s(n) = G_c\, e(n-1). \tag{23}$$

Substituting the above equation into (22), it is possible to obtain

$$d(n) = x(n) + G_c \mathbf{e}_{N+\Delta+1}^T (n-1) \mathbf{f}_{N+\Delta+1} \tag{24}$$

where $\mathbf{e}_{N+\Delta+1}(n) = [e(n)\; e(n-1)\; \cdots\; e(n-N-\Delta)]^T$ and the error signal is given by

$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{s}^c(n-\Delta). \tag{25}$$

Then, (9) becomes

$$\boldsymbol{\epsilon}_2(n) = \mathbf{w}(n) - \mathbf{f}_2 \tag{26}$$

since the adaptive filter will only approximate the last $N+1$ terms of the feedback path impulse response.

If the feedback path is divided into two terms as defined in (10) and (11), relations (23), (24) and (25) can be used to prove that

$$\begin{aligned} e(n) &= x(n) + G_c\, \mathbf{e}_\Delta^T (n-1) \mathbf{f}_{1,\Delta} \\ &- G_c\, \mathbf{e}^T(n-\Delta-1) \boldsymbol{\epsilon}_2^c(n). \end{aligned} \tag{27}$$

The transfer function $H_{ex}^c(z)$ can now be determined as

$$H_{ex}^c(z) = \frac{E(z)}{X(z)} = \frac{1}{D(z)}. \tag{28}$$

where $D(z) = 1 - G_c \mathbf{f}_{1,\Delta}^T\, \mathbf{T}_\Delta(\rho_i)\, \rho_i + G_c\, \boldsymbol{\varepsilon}_2^{cT}\, \mathbf{T}(\rho_i)\, \rho_i^{\Delta+1}$.

Now, the cross-correlation $r_{xe}^c(m)$ can be determined by using the inverse Z-Transform of $\Phi_{xe}^c(z)$ given by

$$\Phi_{xe}^c(z) = \sigma_v^2 H_{xv}(z) H_{xv}(z^{-1}) H_{ex}(z). \tag{29}$$

To compute $r_{ex}^c(m)$ for $m \geq 0$ we note that $r_{ex}^c(m) = \mathcal{Z}^{-1}\{\Phi_{ex}^c(z)\} = \mathcal{Z}^{-1}\{\Phi_{xe}^c(z^{-1})\}$. Using the Residue Theorem [6], it is possible to calculate $r_{ex}^c(m)$

$$r_{ex}^c(m) = \sigma_v^2 \sum_{i=1}^{M} \frac{\rho_i^{M-1}}{\prod_{\substack{j=1 \\ i \neq j}}^{M} (\rho_i - \rho_j)} \frac{1}{\prod_{j=1}^{M} (1 - \rho_i \rho_j)} \frac{1}{D(z)} \rho_i^m \tag{30}$$

The entries for $\mathbf{p}_1^c$ can be determined from (23) and (18) as follows

$$\mathbf{p}_1^c = G_c \mathbf{r}_{ex}^c(\Delta + 1) \tag{31}$$

where the elements for $\mathbf{r}_{ex}^c(\Delta+1)$ are given by (30).

#### 3.1.2. *Autocorrelation Sequence of* $s(m)$

The autocorrelation of $s(m)$ can be calculated from $e(m)$ as follows

$$r_{ss}^c(n) = G_c^2 r_{ee}^c(m). \tag{32}$$

From (27) it is possible to calculate

Table 1: Iterative procedure for calculating $\boldsymbol{\varepsilon}_2^c$

1- Set $\{\boldsymbol{\varepsilon}_2^c\}_0$ = initial guess
2- Calculate $k_{m,j}^c$ using (35) and construct the matrix $\mathbf{K}^c$
3- Calculate $r_{ex}^c(m)$ using (30)
4- Calculate $r_{ee}^c(m)$ using (34)
5- Calculate $\mathbf{p}_1^c$ using (31) and $\mathbf{p}_2^c$ using (19)
6- Calculate $\{\boldsymbol{\varepsilon}_2^c\}_{\mathrm{EST}}$ using (36)
7- If $||\{\boldsymbol{\varepsilon}_2^c\}_{\mathrm{EST}} - \{\boldsymbol{\varepsilon}_2^c\}_0|| \leq \delta_{\min} ||\{\boldsymbol{\varepsilon}_2^c\}_{\mathrm{EST}}||$ : STOP
8- Otherwise $\{\varepsilon_2^c\}_0 \leftarrow \{\varepsilon_2^c\}_{\mathrm{EST}}$ : go to step 2.

$$\begin{aligned} r_{ee}^c(m) &= r_{ex}^c(m) + G_c \mathbf{f}_{1,\Delta}^T \mathbf{r}_{ee,\Delta}^c(m-1) \\ &- G_c \boldsymbol{\varepsilon}_2^{cT} \mathbf{r}_{ee}^m (m - \Delta - 1). \end{aligned} \tag{33}$$

In the above equation, $r_{ex}^c(m)$ is given by (30).

A relationship can be derived for the case where delays are used in the cancellation path

$$\begin{bmatrix} r_{ee}^c(0) \\ \vdots \\ r_{ee}^c(\Delta+1+N) \end{bmatrix} = \alpha \begin{bmatrix} r_{ex}^c(0) \\ \vdots \\ r_{ex}^c(\Delta+1+N) \end{bmatrix}. \tag{34}$$

where $\alpha = [G_c \mathbf{K}^c + \mathbf{I}]^{-1}$. The values of $r_{ex}^c(m)$ can be calculated from (30). The matrix $\mathbf{K}^c$ has elements $k_{m,j}^c$ defined by

$$k_{m,j}^c = \sum_i \varepsilon_{2,i}^c \delta_{(|m-\Delta-1-i|-j)} - \sum_n f_{1,n} \delta_{(|m-1-n|-j)}. \tag{35}$$

For solving the above equation, it is necessary to first determine the elements of the matrix $\mathbf{K}^c$, that are a function of the bias vector elements $\varepsilon_{2,i}^c$.

#### 3.1.3. *Calculation of the bias* $\boldsymbol{\varepsilon}_2^c$

The bias in the adaptive filter estimate, given by (17), is reproduced below as a function of $\mathbf{R}_{ee}^c$, the autocorrelation matrix for $e^c(n)$

$$\boxed{\boldsymbol{\varepsilon}_2^c = \tfrac{1}{G_c^2} [\mathbf{R}_{ee}^c]^{-1}\, \mathbf{p}_1^c + \tfrac{1}{G_c^2} [\mathbf{R}_{ee}^c]^{-1}\, \mathbf{p}_2^c.} \tag{36}$$

The above relation comprises a set of non-linear equations that are not easily solved analytically. It is possible to compute $\boldsymbol{\varepsilon}_2^c$ by using an iterative procedure described in Table 1. The outlined procedure converges to the desired solution in most cases, as will be shown in the next sections. The initial guess can be chosen in a way similar to that described for delays in the forward path [7]. The total bias is then given by (21) then.

## 4. SIMULATIONS

### 4.1. Bias Prediction for a Multi-Pole, Multi-Tap Case

Simulations were performed to verify the outlined numerical method for computing $||\boldsymbol{\varepsilon}_2^c||$. In all simulations, 5 million points and an adaptive filter order of 20 were used with the NLMS algorithm. A small convergence factor of $\mu' = 0.01$ was used to reduce the effect of excess mean-squared error in the simulations. The feedback path was modeled with the first 21 samples of the impulse reponse shown in Fig. 4. The input was a $12^{\text{th}}$-order

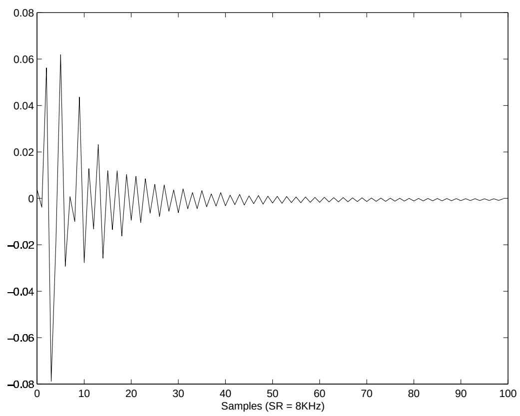


Figure 4: Impulse response of the hearing-aid feedback path for a human subject fitted with a hearing aid [1].

Markov sequence derived from a white Gaussian input that models speech-shaped noise. Theoretical and simulated curves are shown in Fig. 5. The continuous curve represents simulation values and the dashed lines represent values obtained with the procedure outlined in the previous section. It can be seen that for $\Delta < 7$ samples, the theoretical curve is discontinued because the numerical procedure did not converge to the desired solution.

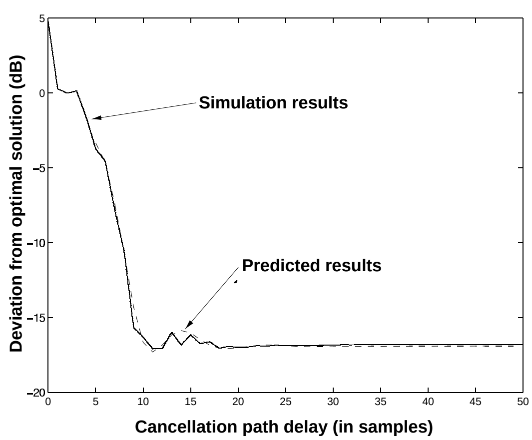


Figure 5: Deviation from optimal solution ($||\boldsymbol{\varepsilon}^c||$) as a function of number of samples in the cancellation path delay for $G_c = 2$. Continuous lines represent simulation results and dashed lines, predicted values. The input was speech-shaped noise ($12^{\mathrm{th}}$ order Markov Process). The feedback path was modeled with the first 21 samples of the impulse response in Fig. 4.

### 4.2. Steady-state performance as a function of delay

The objective of the simulations was to evaluate the steady-state performance of the adaptive filter for various delays. Fig. 6 shows the average convergence metric in the steady-state over a range of delays for four values of the forward path gain $G_c$ (2, 3, 7 and 12). A total of 200,000 samples were used in each simulation. The average convergence metric was obtained by averaging the norm of the difference vector [$||\mathbf{w}(n) - \mathbf{f}||$] over the last 50,000 samples.

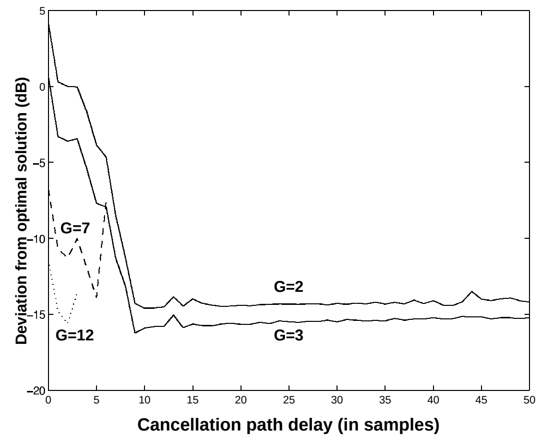


Figure 6: Deviation from optimal solution ($||\boldsymbol{\varepsilon}^c||$) vs. number of samples in the cancellation path delay for $G_c$ = 2, 3, 7, 12. The feedback path used is shown in Fig. 4. The input was speech-shaped noise.

Figure 6 shows the bias as a function of the delay in the cancellation path. The curves for $G_c = 7$ and 12 are discontinuous because the adaptive filter is unstable with a delay greater than 8 and 4 samples, respectively. The reason for the relatively poor performance in this case is that the use of a delay in the cancellation path will approximate the first $\Delta$ samples of the impulse response with zeros - an unreasonable approximation for the feedback path shown in Fig. 4.

To examine the behaviour of the algorithms when a different feedback path is used, simulations were carried out for a case where most of the feedback path impulse response energy is not concentrated in the first samples. A new feedback path impulse response with this property was obtained from measurements performed at the House Ear Institute in Los Angeles. The impulse response of the new feedback path is shown in Fig. 7. Fig. 8 shows that higher gains can be used if the feedback path impulse response does not have most of its energy concentrated in the first samples for a delayed cancellation path.

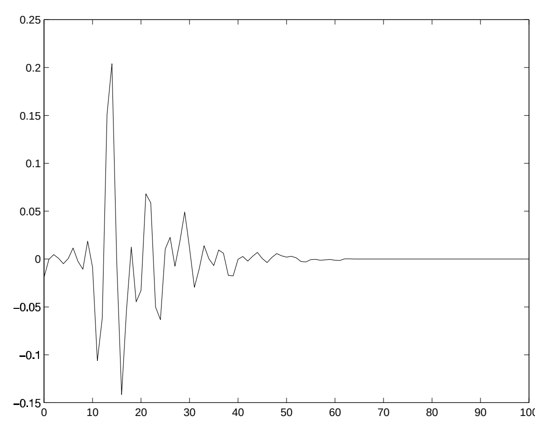

Figure 7: Impulse response of an alternate feedback path.

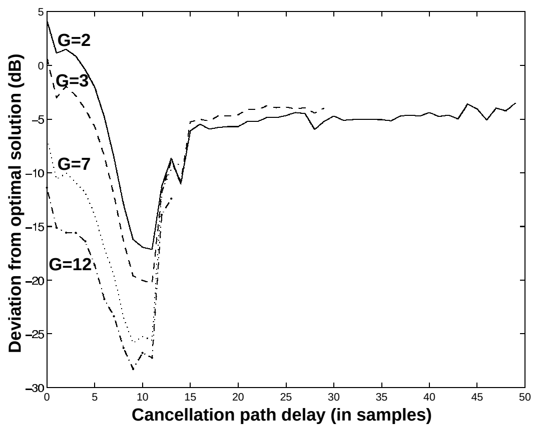


Figure 8: Deviation from optimal solution ($||\boldsymbol{\varepsilon}^c||$) vs. number of samples in cancellation path delay for $G_c$ = 2, 3, 7, 12. The feedback path used is shown in Fig. 7. The input was speech-shaped noise.

### 4.3. The dependence of the bias on the feedback path

As mentioned before, depending on the characteristics of the feedback path, a delay in the cancellation path might prevent the adaptive filter from converging. There is a clear dependency of the adaptive filter performance on the number of samples of delay in the cancellation path. Equations (30) and (33) show that the elements of the vectors $\mathbf{p}_1^c$, $\mathbf{p}_2^c$ and $\mathbf{R}^c$ are dependent on $\mathbf{f}_1$, thus explicitly showing that the bias in this case is a function of the first $\Delta$ elements of the feedback path impulse response.

## 5. SUMMARY AND CONCLUSIONS

Unlike most adaptive filtering applications [5], the LMS algorithm and any algorithm that attempts to approximate a Wiener solution in continuous feedback reduction systems for hearing aids has a bias in its estimate when correlated inputs, such as speech, are used. This bias is due to non-zero cross-correlation of the input and output signals in the hearing aid plant. Analysis for the case of a Markov process as the input signal and delays are used in the cancellation path was presented, and equations that predict the bias were derived.

Simulations indicate that the bias decreases with increasing values of the forward gain $G_c$. Also, the analysis and simulations showed that the bias in the adaptive filter's estimate is a function of the feedback path characteristics. It was shown that, in some cases, the adaptive filter might not obtain a satisfactory approximation of the feedback path, causing the system to become unstable for high values of the forward gain $G_c$, as indicated in Fig. 6. This result can be justified by observing that when delays are used in the cancellation path, they approximate the first $\Delta$ samples of the impulse response with zeros, causing the resulting bias to be dependent on the values of those samples. This approximation might or might not be accurate, depending on the characteristics of the feedback path. This dependency on the feedback path is in contrast with our previous study [7] that analyzed the effect of delays in the forward path; it was shown that the bias in that case is independent of the feedback path. Finally, simulations and theoretical curves illustrated in Figs. 5-8 clearly show that the smallest value of delay that minimizes the bias significantly is around 10 samples (1.25 ms); this value is the same as that chosen by Bustamante et al [3] experimentally.

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