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Homework assignment #2

The steepest descent algorithm and nondifferentiable functions.

1. Let $\gamma > 1$. Show that the function

$$f(x_1, x_2) = \begin{cases} \sqrt{x_1^2 + \gamma x_2^2} & |x_2| \leq x_1 \\ \dfrac{x_1 + \gamma |x_2|}{\sqrt{1+\gamma}} & \text{otherwise} \end{cases}$$

   is convex. You can do this, for example, by verifying that

$$f(x_1, x_2) = \sup \left\{ x_1 y_1 + \sqrt{\gamma} x_2 y_2 \;\middle|\; y_1^2 + y_2^2 \leq 1,\ y_1 \geq 1/\sqrt{1+\gamma} \right\}.$$

   Note that f is unbounded below. (Take $x_2 = 0$ and let x_1 go to $-\infty$.)

2. Consider the steepest descent algorithm applied to f, with starting point $x^{(0)} = (\gamma, 1)$, and using an exact line search. Show that the iterates are

$$x_1^{(k)} = \gamma \left(\frac{\gamma - 1}{\gamma + 1} \right)^k, \qquad x_2^{(k)} = \left(-\frac{\gamma - 1}{\gamma + 1} \right)^k.$$

   Therefore $x^{(k)}$ converges to $(0, 0)$. However, this is not the optimum, since f is unbounded below.