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Homework assignment #3

If C is a closed convex set then

$$Q_t(u,v) = \mathop{\rm argmin}_{z\in C}\left(tu^Tz + \frac{1}{2}\|z-v\|_2^2\right) = \mathop{\rm argmin}_{z\in C}\|z-(v-tu)\|_2^2$$

is the Euclidean projection of $v-tu$ on C. Derive closed-form expressions or simple algorithms for the following generalized projections.

1. The projection

   $$Q_t(u,v) = \mathop{\rm argmin}_{z\in C}\left(tu^Tz + d(z,v)\right)$$

   on the unit simplex, using the relative entropy distance:

   $$C = \{z \mid z \succeq 0, \mathbf{1}^Tz \le 1\}, \qquad d(z,v) = \sum_{i=1}^n (z_i\log(z_i/v_i) - z_i + v_i).$$

   We assume $v \succ 0$.

2. The projection

   $$Q_t(u,v) = \mathop{\rm argmin}_{z\in C}\left(tu^Tz + d(z,v)\right)$$

   on the nonnegative orthant, using the sum of the quadratic distance and the Bregman distance derived from the logarithmic barrier function:

   $$C = \mathbf{R}^n_+, \qquad d(z,v) = \frac{1}{2}\|z-v\|_2^2 + \sum_{i=1}^n (z_i/v_i - \log(z_i/v_i)) - n.$$

   We assume $v \succ 0$.

3. The projection

   $$Q_t(U,V) = \mathop{\rm argmin}_{Z\in C}\left(t\,\mathbf{tr}(UZ) + d(Z,V)\right)$$

   on the positive semidefinite cone, using the sum of the quadratic distance and the Bregman distance derived from the log-determinant barrier function:

   $$C = \mathbf{S}^n_+, \qquad d(Z,V) = \frac{1}{2}\|Z-V\|_F^2 + \mathbf{tr}(ZV^{-1}) - \log\det(ZV^{-1}) - n.$$

   We assume $V \succ 0$.