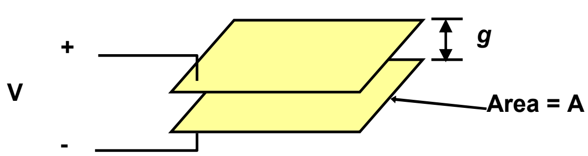
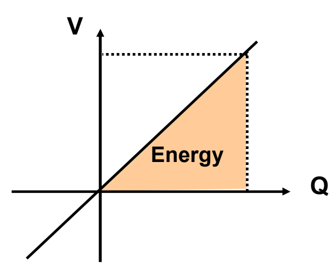
Electrostatic Actuators (1)

Reading: Chapter 6 of Senturia

Parallel Plate Capacitor





$C = \frac{\varepsilon A}{g}$ **Capacitance**

A: **Area**

g: **Gap spacing**

ε: **Permittivity**

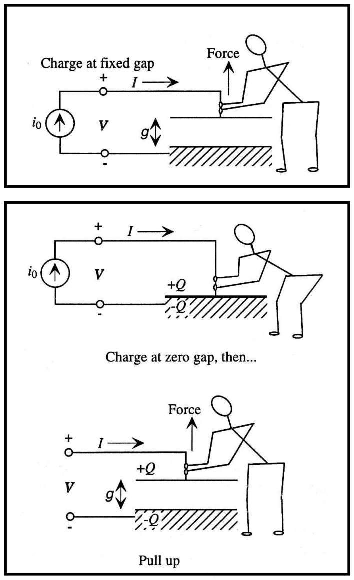
$Q = CV$

$W(Q) = \frac{Q^2}{2C} = \frac{Q^2 g}{2\varepsilon A}$

Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ **F/m**

Energy Stored in a Capacitor

2 ways to charge the capacitor:



Energy stored in a capacitor is a function of total charge, Q, and gap spacing, g

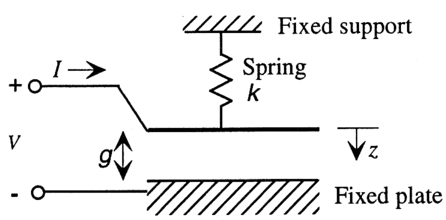
$$W(Q,g)=\frac{Q^2 g}{2\varepsilon A}$$

Energy change:

$$dW = Fdg + VdQ$$

Mechanical Work (Fdg) — **Electrical Work** (VdQ)

Electrostatic Gap Closing Actuator (1) Charge Control



$$F = \left.\frac{\partial W(Q,g)}{\partial g}\right|_Q = \frac{Q^2}{2\varepsilon A}$$

Electrostatic force is
- **independent of g**
- **proportional to the square of the charge**

At equilibrium, electrostatic force is balanced by elastic force:

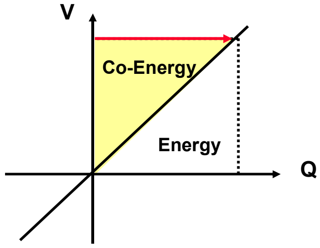
$$F_{electrostatic} = F_{elastic}$$

$$\Rightarrow \frac{Q^2}{2\varepsilon A} = kz$$

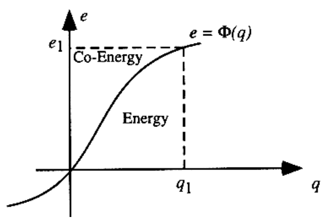
z: displacement of the movable plate

$$\Rightarrow z = \frac{Q^2}{2\varepsilon A k}$$

Electrostatic Gap Closing Actuator (2) Voltage Control



Linear Capacitor



Nonlinear Capacitor

Co-Energy:

$$W^*(V,g) = QV - W(Q,g)$$
$$\Downarrow$$
$$dW^*(V,g) = (QdV + VdQ) - dW(Q,g)$$
$$= (QdV + VdQ) - (Fdg + VdQ)$$
$$\Downarrow$$
$$dW^*(V,g) = QdV - Fdg$$

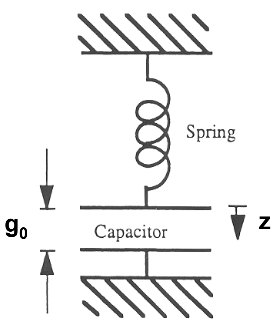
Force:

$$F = -\left.\frac{\partial W^*(V,g)}{\partial g}\right|_V = \frac{\varepsilon A V^2}{2g^2}$$

Electrostatic force with voltage control is

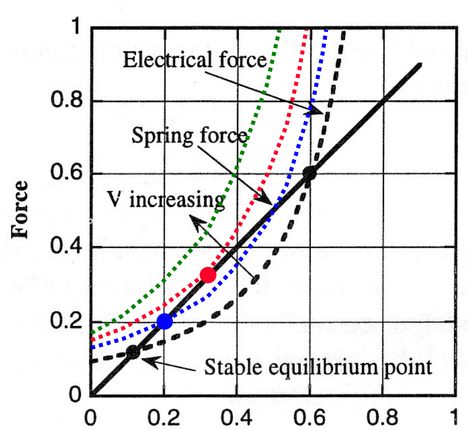
- **inversely proportional to g^2**
- **proportional to V^2**

Graphical Solution for Voltage Controlled Gap Closing Actuator



$$F_{elec} = \frac{\varepsilon_0 V^2}{2} \cdot \frac{A}{(g_0 - z)^2}$$

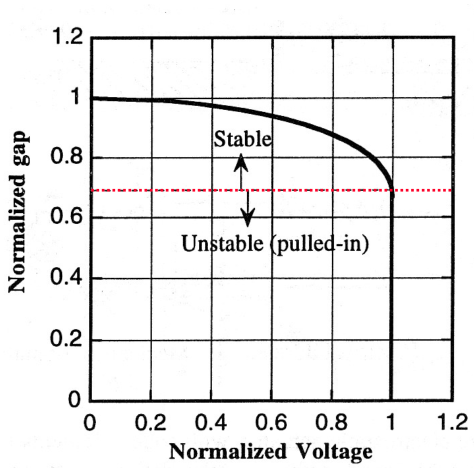
$$F_{spring} = k \cdot z$$

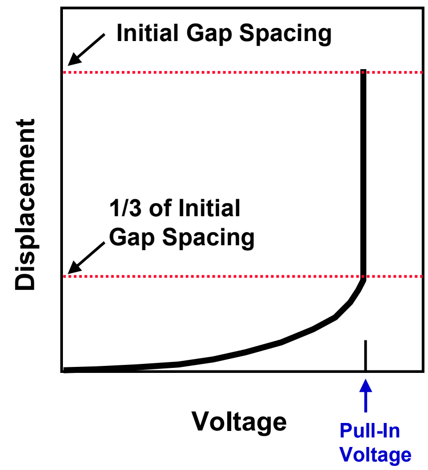


$$\varsigma = \frac{z}{g_0} = 1 - \frac{g}{g_0}$$

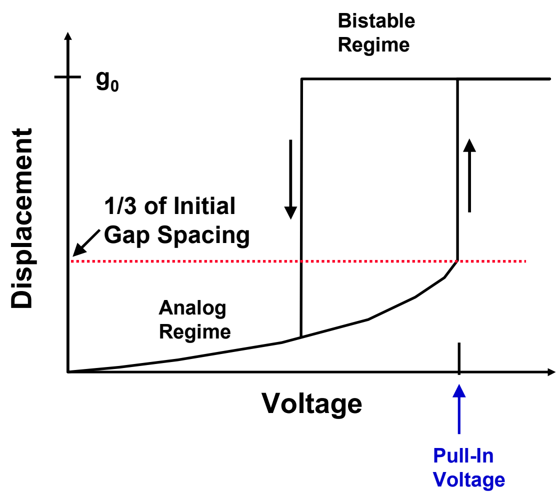
Normalized Displacement

Transfer Curve of Gap Closing Actuators





Bistability



Stability Analysis

- An equilibrium point is unstable if in the presence of small perturbation, the net force does not tend to return it back to equilibrium position

- For stable point, the variation of the net force should be of opposite sign with the perturbation.

$$\frac{\delta F_{net}}{\delta g} < 0$$

$$F_{net} = -\frac{\varepsilon A V^2}{2g^2} + k(g_0 - g)$$

$$g \rightarrow g + \delta g$$

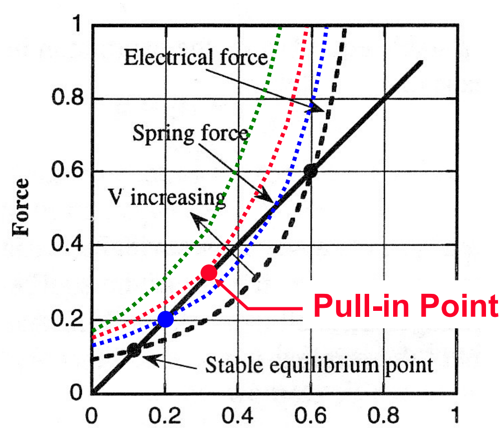
$$\delta F_{net} = \left.\frac{\partial F_{net}}{\partial g}\right|_V \delta g = \left(\frac{\varepsilon A V^2}{g^3} - k\right)\delta g$$

$$\Rightarrow \frac{\varepsilon A V^2}{g^3} \le k$$

$$V \le \sqrt{\frac{kg^3}{\varepsilon A}}$$

(Please note: *g* is the instantaneous gap spacing, not initial gap spacing)

Pull-in Voltage



$$\varsigma = \frac{z}{g_0} = 1 - \frac{g}{g_0}$$

Normalized Displacement

- **At pull-in point:**
  - **Net force is zero:**

    $$F_{net} = 0$$
  - **On the verge of becoming unstable**

    $$\frac{\delta F_{net}}{\delta g} = 0$$
  - **2 equations, 2 variables (*g* and *V*)**

- **Solution:**

$$g_{PI} = \frac{2}{3} g_0 \qquad V_{PI} = \sqrt{\frac{8kg_0^3}{27\varepsilon A}}$$

The parallel plate electrostatic actuator becomes unstable for V > V_{PI}