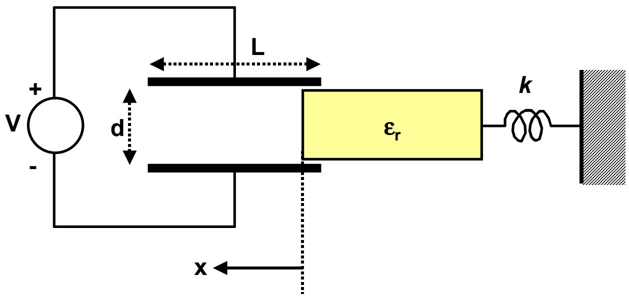
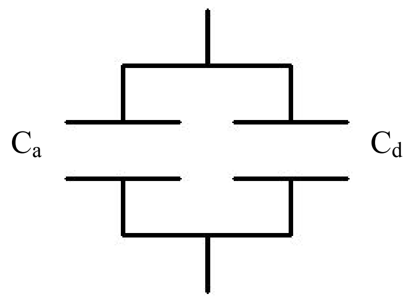
Solution for Homework #1



V : Voltage, L : Length, W : Width, d : the distance between plates, ε : Permittivity, K : Spring constant

(a)This problem can be considered as two parallel capacitors:



$$C_a = \frac{\varepsilon_o A_a}{d} = \frac{\varepsilon_o A}{d}\frac{L-x}{L}$$

$$C_d = \frac{\varepsilon_o \varepsilon_r A_d}{d} = \frac{\varepsilon_o \varepsilon_r A}{d}\frac{x}{L}$$

Co-energy

$$W^*(V,x) = \frac{1}{2}CV^2 = \frac{1}{2}(C_a + C_d)V^2 = \frac{1}{2}\frac{\varepsilon_o A}{d}\left[1+(\varepsilon_r - 1)\frac{x}{L}\right]V^2$$

For the case of voltage control

$$F_{electro} = \frac{\partial W^*(V,x)}{\partial x} = \frac{1}{2}\frac{\varepsilon_o A}{d}(\frac{\varepsilon_r - 1}{L})V^2$$

(b)

$$F_{net} = F_{electro} + F_{spring} = \frac{1}{2}\frac{\varepsilon_o A}{d}(\frac{\varepsilon_r - 1}{L})V^2 - kx$$

$$\delta F_{net} = \left.\frac{\partial F_{net}}{\partial x}\right|_V \delta x = -k\delta x$$

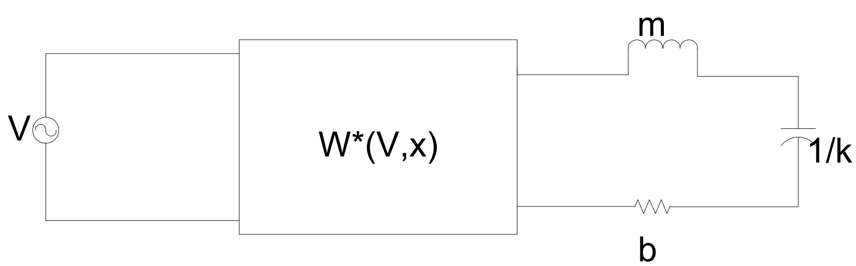
Because $-k < 0$, the system is always stable. There is no pull in phenomenon.

(c)

$$F_{electro} = F_{spring}$$

$$x = \frac{1}{2}\frac{\varepsilon_o A}{dk}(\frac{\varepsilon_r - 1}{L})V^2$$

(d) Equivalent lumped circuit element model



(e) Transfer Function

From lumped circuit element model

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

Take Laplace Transform of the above equation

$$X(s) = \frac{1}{ms^2 + bs + k}F(s)$$

$\frac{1}{ms^2 + bs + k}$ →Transfer Function

Assume we our input is a constant V

$$F(s) = \frac{1}{2}\frac{\varepsilon_o A}{d}(\frac{\varepsilon_r - 1}{d})V^2\frac{1}{s}$$

* Note: $(\frac{1}{s})$ is the Laplace transform of the step function

$$\Rightarrow X(s) = \frac{1}{s(ms^2 + bs + k)} \times \frac{1}{2}\frac{\varepsilon_o A}{d}(\frac{\varepsilon_r - 1}{d})V^2$$

To find the steady state displacement, the Final-Value is used

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \frac{1}{2}\frac{\varepsilon_o A}{dk}(\frac{\varepsilon_r - 1}{d})V^2$ -----same as the result in (b)

(f) V=10 volt , L=500μm , W=500μm , ε_r=40 , k=1N/m , and assume d=100μm
Plug the values in

$$F_{electro} = \frac{1}{2}\frac{\varepsilon_o A}{d}(\frac{\varepsilon_r - 1}{L})V^2 \rightarrow F_{electro} = 8.6 \times 10^{-8} N$$

$$x = \frac{1}{2}\frac{\varepsilon_o A}{dk}(\frac{\varepsilon_r - 1}{L})V^2 \rightarrow x = 8.6 \times 10^{-8} m$$