EE M250B/ MAE M282 / BME M250B
HW2 solution

(a) The equation of motion is described as

$$T(t) = I\ddot{\theta}(t) + C\dot{\theta}(t) + K\theta(t) \quad (1)$$

where T, I, C, θ, and K represent torque, moment of inertia, damping coefficient, rotation angle, and tortional stiffness, respectively.

On the other hand, the driving torque by this actuator is described as

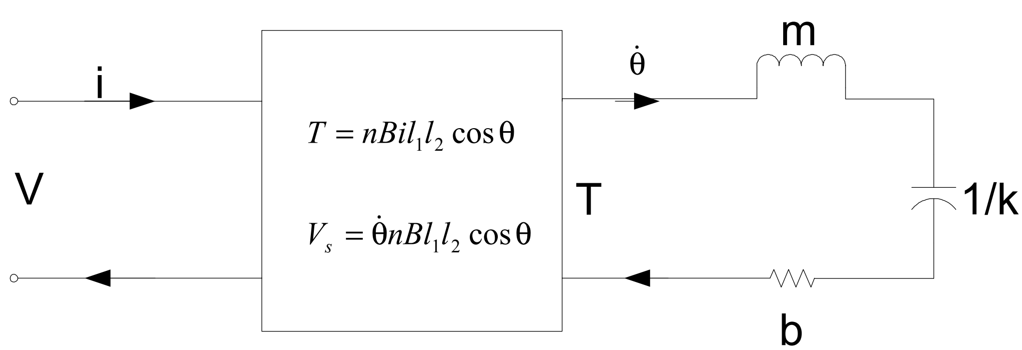
$$T(t) = nBl_1l_2\, i(t)cos\theta \quad (2)$$

where n, i, B, l_1, l_2 , and θ represent number of coil turns, current, magnetic flux density, length of coil adjacent to the magnets, and distance between coils adjacent to the magnets, respectively.

$$T_{spr} = k\theta$$

$$\frac{i(t)}{\theta \sec\theta} = \frac{k}{nBl_1l_2}$$

(b) The equivalent circuit



Using Kirchoff's Law , the mechanical part can be represented by the following equation

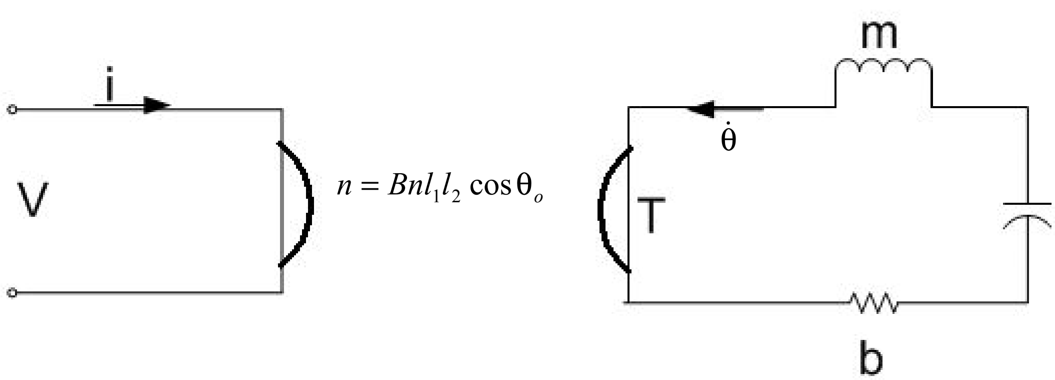
$$m\ddot{\theta} + b\dot{\theta} + k\theta = nBil_1l_2\cos\theta$$

(c) This is a nonlinear equation, we can not use Laplace transform to solve it. This equation can be simulated by Simulink like the example of magnetic actuator, which we discussed in class. In question (a), we only consider the force equilibrium between the Lorenz Force and the spring force. This is static equilibrium. That means the system is in a steady state and $\dot{\theta}$ and $\ddot{\theta}$ equal to zero because of the damping b. To compare with the solution in (a), we can neglect terms of $\dot{\theta}$ and $\ddot{\theta}$ and only consider the Lorenz force and the force from the spring. The relationship between the input current and output angle is

$$\frac{i(t)}{\theta \sec\theta} = \frac{k}{nBl_1 l_2}$$

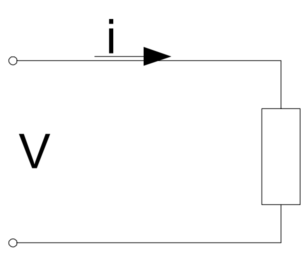
which is exactly the same as question (a).

If we consider a small angle actuation at a specific angle θ_o, the system can be approximated to a linear system. The nonlinear two-port element can be replaced by a gyrator. The equivalent circuit can be represented by



$$\theta_o : operation \quad point$$

$$\begin{pmatrix} \delta T \\ \delta\dot{\theta} \end{pmatrix} = \begin{pmatrix} 0 & nBl_1 l_2 \cos\theta_o \\ -\frac{1}{nBl_1 l_2 \cos\theta_o} & 0 \end{pmatrix} \begin{pmatrix} \delta V \\ \delta I \end{pmatrix}$$



$$Z_{in}(s) = \frac{n^2}{Z_{out}(s)} = \frac{(nBl_1 l_2 \cos\theta_o)^2}{ms^2 + bs + k}$$

By using Kirchoff's Laws, we can find the relationship between current and sensing voltage. Once we get the sensing voltage, the angular velocity can be found by dividing voltage by n. Remember, this linearization analysis is only valid for small-signal analysis, i.e., small angle rotation in specific operation point. Once the rotation angle becomes large, n is not a constant but a function of rotation angle, in which case the two-port gyrator element become nonlinear.

(d) The relation between torque T and angle θ is

$$T = K\frac{G}{L}\theta = k\theta$$

T : twisting moment
θ : angle of twist
L : length of one torsion beam
G :modulus of rigidity (or Shear Modulus)
K : shape/dimension factor

So the compliance of one torsion beam is

$$c = \frac{1}{k} = \frac{L}{KG} = \frac{500\times10^{-6}}{5\times10^{-24}\times[\frac{16}{3} - 3.36\times\frac{1}{5}(1-\frac{1}{12\times5^4})]\times\frac{166\times10^9}{2(1+0.28)}} = 3.31\times10^8(rad/N\cdot m)$$

(100) silicon
$E = 166GPa, \upsilon = 0.28$

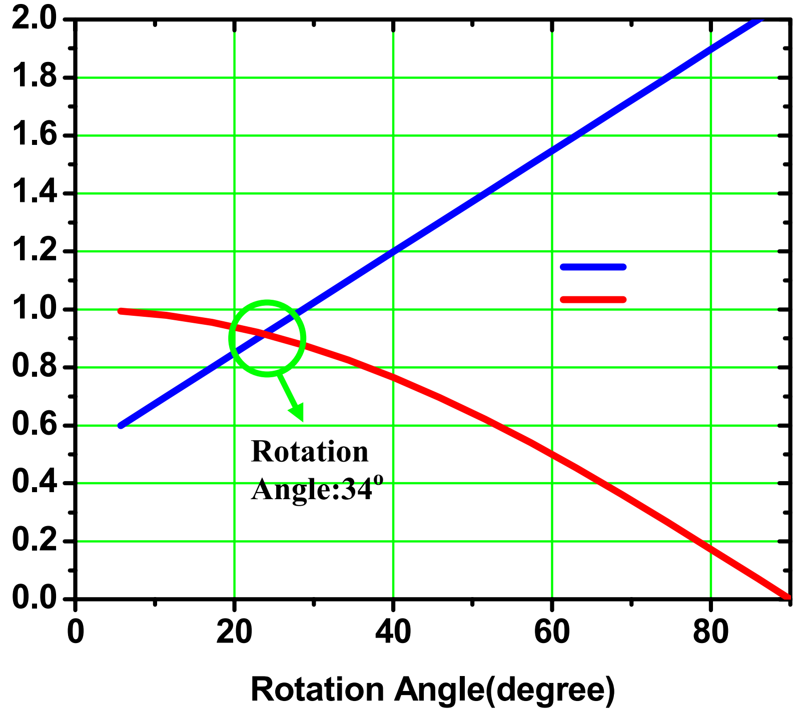
Since there are two torsion beams, the compliance of the system is

$$c_{total} = \frac{1}{2k} = 1.65\times10^8(rad/N\cdot m)$$

(e) The torque generated by electromagnetic force is equal to the torque of the two torsion beams.
Angular displacement

$$BIW^2\cos\theta = 2k\theta$$

$$\frac{\theta}{\cos\theta} = \frac{1}{2}BIW^2\frac{1}{k} = 0.5\times1\times10\times10^{-3}\times(100\times10^{-6})^2\times3.31\times10^8 = 1.65$$



2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
Rotation Angle(degree)
Rotation
Angle:34°