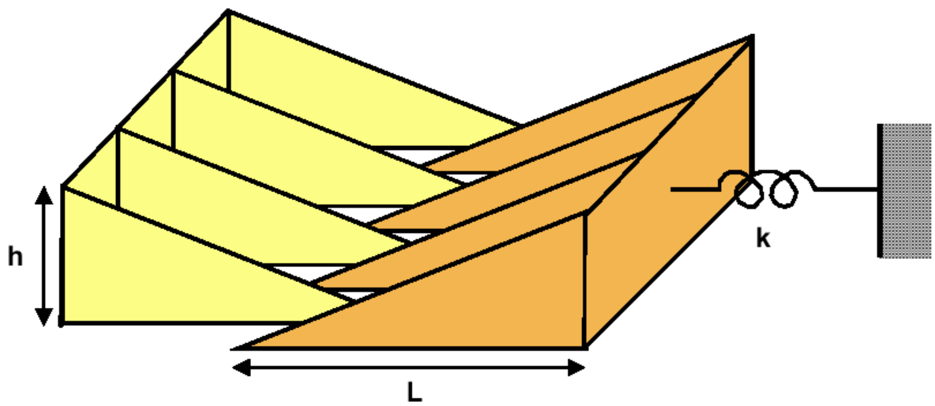
# EE M250B / MAE M282 / BME M250B
# HW#3 Solution




**(a)**

Given: $\tan\theta = h/L$

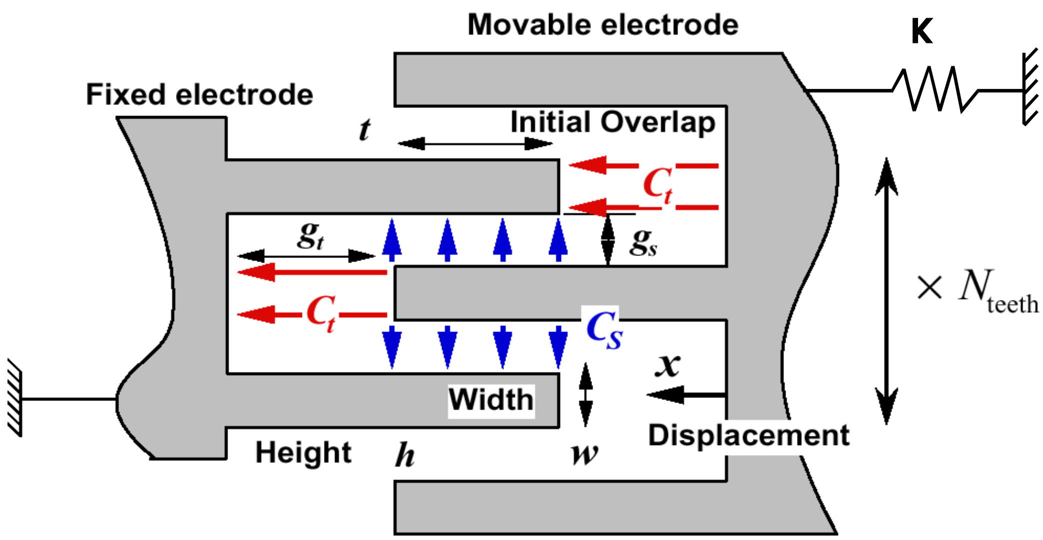
Assume that

(1) the initial overlap length: t

(2) the displacement of the movable comb: x

(3) we can neglect the fringe field.

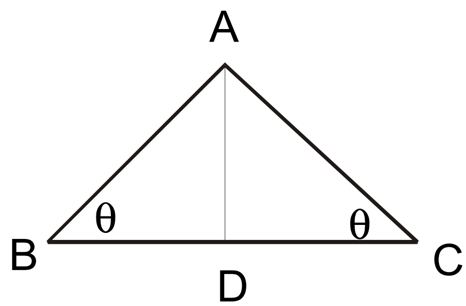
For the schematic view of the comb drive shown below:




The capacitance between the finger's tips: $C_t \approx 0$

The capacitance between the sides of the fingers: $C_s = 2\frac{\varepsilon_0 A}{g}$

For a given displacement x, the overlap area is the triangle as shown below:




Where

$$\overline{BC} = t + x$$

$$\overline{AD} = \frac{1}{2}\overline{BC}\tan\theta = \frac{h}{2L}(t+x)$$

Overlap area

$$A = \frac{1}{2}\overline{BC}\times\overline{AD} = \frac{h}{4L}(t+x)^2$$

Therefore, we can get $C_s$:

$$C_s = 2\frac{\varepsilon_0}{g}(\frac{h}{4L})(t+x)^2 = \frac{\varepsilon_0 h}{2gL}(t+x)^2$$

Total capacitance of N gaps C:

$$C = N\times C_s = \frac{\varepsilon_0 hN}{2gL}(t+x)^2$$

So, the electrostatic energy stored in the triangular comb drive is:

$$W^* = \frac{1}{2}CV^2 = \frac{\varepsilon_0 hN}{4gL}V^2(t+x)^2$$

**(b)** The electrostatic force: (in x-direction)

$$F_e = \frac{\partial W^*}{\partial x} = \frac{1}{2}V^2\frac{\partial C}{\partial x} = \frac{1}{2}V^2\frac{\partial}{\partial x}\left[\frac{\varepsilon_0 hN}{2gL}(t+x)^2\right] = V^2(\frac{\varepsilon_0 hN}{2gL})(t+x)$$

**(c)**

$$F_{net} = F_e - F_m = V^2(\frac{\varepsilon_o hN}{2gL})(t+x) - kx,$$

For small perturbation $\delta x$,

$$\delta F_{net} = \left(\frac{\partial\ Fnet}{\partial x}\right)_V * \delta x = \left(V^2 \times \frac{\varepsilon_o hN}{2gL} - k\right)\delta x$$

When

$\left(V^2 \times \frac{\varepsilon_o hN}{2gL} - k\right) \le 0$ ===➔ Stable

That is, $V^2 \le \frac{2gLk}{\varepsilon_o hN}$, or $V \le \sqrt{\frac{2gLk}{\varepsilon_o hN}} = V_{PI}$.

So, for the triangular comb drive, we will have pull-in if $V > \sqrt{\frac{2gLk}{\varepsilon_o hN}}$,

**(d)**

Electrostatic force of N fingers:

$$F_e = V^2 \times \left(\frac{\varepsilon_0 hN}{2gL}\right)(t + x)$$

Mechanical spring force:

$$F_m = kx$$

For equilibrium: $F_e = F_m$, we can get:

$$V^2 \times \left(\frac{\varepsilon_0 hN}{2gL}\right)(t + x) = kx$$

The transfer function of displacement and driving voltage is:

$$x = \frac{V^2\left(\frac{\varepsilon_0 hNt}{2gL}\right)}{k - V^2\left(\frac{\varepsilon_0 hNt}{2gL}\right)}$$

**(e)**

For a poly-Si folded beam:

$w_s$=2µm , $t_s$=2µm, $l_s$=100µm, E=160GPa

The spring constant of the pair of folded springs is:

$$k = 2 \times \frac{6EI}{l_s^3} = \frac{E w_s t_s^3}{l_s^3} = \frac{160\times10^9 \times 2\times10^{-6} \times (2\times10^{-6})^3}{(100\times10^{-6})^3} = 2.56 \text{ N/m}$$

**(f)**

For h = 10μm, L=50μm, g=2μm, V=20 volt, k=1.28 N/m
assume that N=100 and initial overlap t= 10μm
Plug into the equation from (d)

$$V^2 = \frac{2gLkx}{\varepsilon_0 hN(t+x)}$$

$$20^2 = \frac{2\times 2\times 10^{-6}\times 50\times 10^{-6}\times 1.28\times x\times 10^{-6}}{8.85\times 10^{-12}\times 10\times 10^{-6}\times 100\times (10\times 10^{-6}+x\times 10^{-6})}$$

displacement : $x = 0.14$ μm

actuating force: F= *kx* = $1.28\times 0.14\times 10^{-6} = 1.79\times 10^{-7}$ N