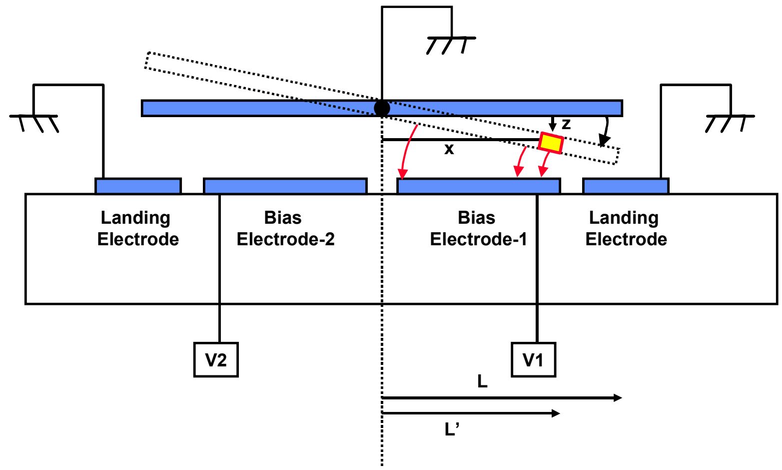
EE M250B / MAE M282 / MBE M250B
HW#5 Solution



Variable definition:
z_o: initial gap
θ: rotation angle of mirror
θ_M: Maximum rotation angle (restrained by mechanical stopper)
w: mirror width

The mirror can be considered as many small parallel capacitors connected parallel
Force generated by the capacitor between the x and x+dx equals to

$$dF = \left.\frac{\partial(dW^*(V,g))}{\partial g}\right|_V = \frac{\varepsilon V^2}{2g^2} w dx$$

Integrate dF from 0 to L', we can get the total torque

$$\tau_a = \int_0^{L'} x \frac{\varepsilon V^2}{2(z_o - z)^2} w dx$$

Let $g = z_o - z \qquad \tan\theta = \frac{z}{x} \qquad \tan\theta_M = \frac{z_o}{L}$

$$\tau_a = \frac{\varepsilon V^2 W}{2} \int_0^{L'} \frac{x dx}{(z_o - x\tan\theta)^2}$$

Use the technique of integration by parts. Let

$$u = x$$
$$du = dx$$
$$v = \frac{1}{\tan\theta(z_o - x\tan\theta)}$$
$$dv = \frac{x}{(z_o - x\tan\theta)^2}$$

$$\tau_a = \frac{\varepsilon V^2 W}{2}\left\{\left[\frac{x}{\tan\theta(z_o - x\tan\theta)}\right]_0^{L'} - \frac{1}{\tan\theta}\int_0^{L'}\frac{dx}{z_o - x\tan\theta}\right\}$$
$$\tau_a = \frac{\varepsilon V^2 W}{2}\left\{\left[\frac{x}{\tan\theta(z_o - x\tan\theta)}\right]_0^{L'} - \frac{1}{\tan\theta}\left[-\frac{1}{\tan\theta}\ln(z_o - x\tan\theta)\right]_0^{L'}\right\}$$
$$\tau_a = \frac{\varepsilon V^2 W}{2\tan\theta}\left[\frac{x}{z_o - x\tan\theta} + \frac{1}{\tan\theta}\ln(z_o - x\tan\theta)\right]_0^{L'}$$
$$\tau_a = \frac{\varepsilon V^2 W}{2\tan\theta}\left[\frac{L'}{z_o - L'\tan\theta} + \frac{1}{\tan\theta}\ln(z_o - L'\tan\theta) - \frac{1}{\tan\theta}\ln(z_o)\right]$$
$$\tau_a = \frac{\varepsilon V^2 W}{2\tan^2\theta}\left[\ln\left(\frac{z_o - L'\tan\theta}{z_o}\right) + \frac{L'\tan\theta}{z_o - L'\tan\theta}\right]$$

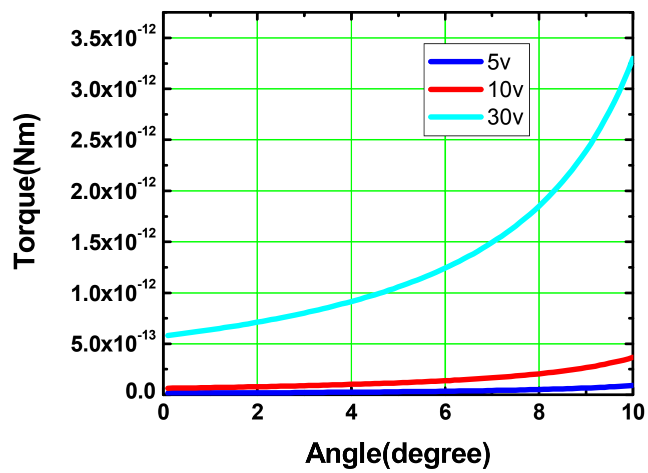
Let $z_o = L\tan\theta_M$, $\beta = \frac{L'}{L}$

$$\tau_a = \frac{\varepsilon V^2 W}{2\tan^2\theta}\left[\ln\left(\frac{L\tan\theta_M - \beta L\tan\theta}{L\tan\theta_M}\right) + \frac{\beta L\tan\theta}{L\tan\theta_M - \beta L\tan\theta}\right]$$
$$\tau_a = \frac{\varepsilon V^2 W}{2\tan^2\theta}\left[\ln\left(1 - \frac{\beta\tan\theta}{\tan\theta_M}\right) + \frac{\beta\tan\theta}{\tan\theta_M - \beta\tan\theta}\right]$$

Let $\alpha = \frac{\tan\theta}{\tan\theta_M}$

$$\tau_a = \frac{\varepsilon V^2 W}{2\tan^2\theta}\left[\ln(1-\alpha\beta) + \frac{\alpha\beta\tan\theta}{\tan\theta - \alpha\beta\tan\theta}\right]$$
$$\tau_a = \frac{\varepsilon V^2 W}{2\tan^2\theta}\left[\ln(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\right]$$
$$\tau_a = \frac{\varepsilon V^2}{2}\cdot\frac{W}{\tan^2\theta_M}\left[\frac{1}{\alpha^2}\ln(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\right]$$

(b)



(c) Compliance of the spring

$$C = \frac{1}{k}$$

$$k = \frac{KG}{L} \qquad G = \frac{E}{2(1-\nu)} \qquad K : ab^3\left(\frac{16}{3} - 3.36\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right)$$

Where a=0.5width=0.5um , b=0.5thickness=0.03um , E=70Gpa , ν=0.32 , L=6um

Then

$$C = 1.68\times 10^9 \text{ (rad/Nm)}$$

(d) Pull-in: When voltage above 212V, there is no intersection between the spring force and electrostatic force. Pull-in happens when under this condition.