2003 EE M250B Midterm Solution

(1) $Gpa := 10^9$ $\mu m := 10^{-6}$ $\varepsilon 0 := 8.854\cdot 10^{-12}$ $\rho := 2331$

$E := 160\cdot Gpa$

(a) In surface micromachining, we need a ground plane (which provides electrical interconnect), a sacrificial layer, and a structural layer. We need three photomasks, one for ground plane, another one for sacrifical layer, and the third one for the mechanical structures.

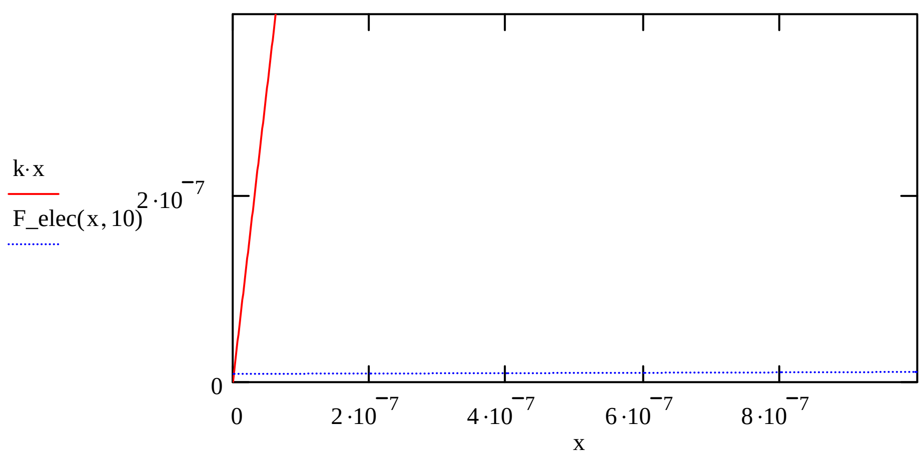
(b) On SOI, we can use the buried oxide as sacrificial layer, and the single crystal silicon (SCS) as the structural layer. The substrate provides the ground plane. The structures will be released by time etching in HF. So we need to modify the structures such that all the suspended structures are narrower than the anchor parts. The solid parts in the figure needs to be replaced by "skeleton" or "grid" structures. Only one photomask is needed.

(c) $t := 20\cdot\mu m$ $L := 100\cdot\mu m$ $g := 10\cdot\mu m$ $w := 2\cdot\mu m$ $S := 200\cdot\mu m$

$$F\_elec(x, V) := \frac{\varepsilon 0\cdot V^2}{2}\cdot\frac{t\cdot L}{(g-x)^2}$$

Spring constant: $I := \frac{t\cdot w^3}{12}$

$$k := 4\cdot\frac{6\cdot E\cdot I}{S^3}$$ $k = 6.4$ N/m



Initial value: $x := 0.1\cdot\mu m$

Given

$F\_elec(x, 10) = k\cdot x$

$x0 := Find(x)$ $\frac{x0}{\mu m} = 1.383\times 10^{-3}$

(Note: This is a third-order polynomial, and may not be easily solved by hand. So as long as you write down the equation for teh displacement, you will get the point.)

(d) $V\_pullin := \sqrt{\dfrac{8 \cdot k \cdot g^3}{27 \cdot \varepsilon 0 \cdot t \cdot L}}$ $V\_pullin = 327.241$ volts

(e) (i) If the minimum width (i.e., width for spring) is the same, since both electrostatic force and the spring constant are linearly proportional to t, the thickness of SOI, the pull-in voltage should remain the same.
(ii) If the spring width is limited by the aspect ratio of 10:1, the spring width will become 100/10 = 10 um. The spring constant will increase by (10/2)^3=125, so the pull-in voltage increases by a factor of

$$\sqrt{5^3} = 11.18$$

3)

(a) $\mu 0 := 4 \cdot \pi \cdot 10^{-7}$ $\mu core := 1000$ $E := 100 \cdot Gpa$

$L\_core := 3 \cdot 1 \cdot 10^{-3}$ $A\_core := 100 \cdot \mu m \cdot 100 \cdot \mu m$ $R\_core := \dfrac{L\_core}{\mu 0 \cdot \mu core \cdot A\_core}$

$L\_beam := 1 \cdot 10^{-3}$ $A\_beam := 100 \cdot \mu m \cdot 100 \cdot \mu m$ $R\_beam := \dfrac{L\_beam}{\mu 0 \cdot \mu core \cdot A\_beam}$

$L\_gap := 10 \cdot \mu m$ $A\_gap := 100 \cdot \mu m \cdot 100 \cdot \mu m$ $R\_gap(x) := \dfrac{L\_gap - x}{\mu 0 \cdot A\_gap}$

$$F(Ni, x) := \frac{1}{2} \cdot \frac{2Ni^2}{\mu 0 \cdot A\_gap \cdot (R\_core + R\_beam + 2\,R\_gap(x))^2}$$

$R\_core = 2.387 \times 10^8$ $R\_beam = 7.958 \times 10^7$ $R\_gap(0) = 7.958 \times 10^8$

$R\_gap(5 \cdot \mu m) = 3.979 \times 10^8$

The Reluctances of the Core and Gap parts are actually comparable (because the length of the core is much longer)

(b) Yes, it has pull-in since the force expression is similar to that of electrostatic gap-closing actuators. The simplest way to solve to neglect the reluctance in the core, then the force expression is similar to that for gap closing actuator. You can get the pull-in current by substituting $\varepsilon 0$ with $\mu 0$ in the pull-in voltage equation (see Prob. (1)(d) solution above), and divide the results by N to obtain current.

$t := 100 \cdot \mu m$ $L := 700 \cdot \mu m$ $w := 5 \cdot \mu m$

Spring constant: $I := \dfrac{t \cdot w^3}{12}$

$k := 2 \cdot \dfrac{6 \cdot E \cdot I}{L^3}$ $k = 3.644$ N/m

$I := \dfrac{1}{100} \cdot \sqrt{\dfrac{8 \cdot k \cdot L\_gap^3}{27 \cdot \mu 0 \cdot A\_gap}}$ $I = 2.931 \times 10^{-3}$

The exact numerical solution is given below. It includes the reluctance of the core, which in this case is not completely negligible. So the resulting current is slightly larger than the simplified solution above.

Initial assumption: $Ni := 100$ $x := 10\cdot\mu m$

Given $F(Ni, x) = k\cdot x$

Pull-in Condition

$$\frac{d}{dx}F(Ni, x) = k$$

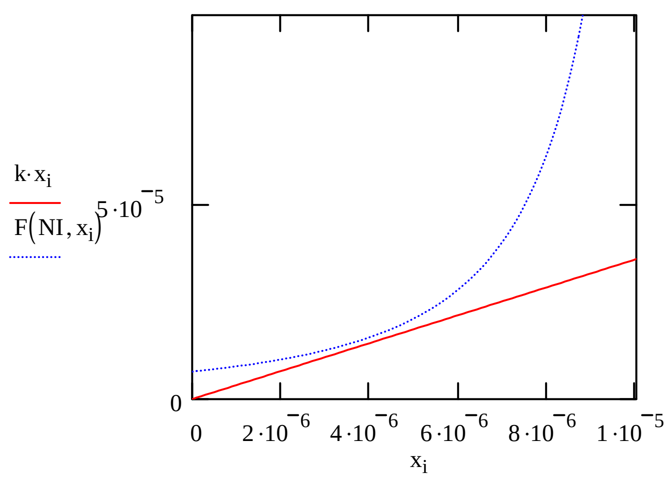
$$\begin{pmatrix} NI \\ x0 \end{pmatrix} := \text{Find}(Ni, x)$$

$NI = 0.573$ $x0 = 3.732\times 10^{-6}$ Pull-in position

$I := \frac{NI}{100}$ $I = 5.725\times 10^{-3}$ Amp

$i := 0..100$ $x_i := \frac{i}{100}\cdot L\_gap$

Verification by
Graphic solution:



(c) The force is larger since the reluctance of the gap becomes smaller.

$L\_gap := 10\cdot\mu m$ $A\_gap := 20\cdot\mu m\cdot 20\cdot\mu m$ $R\_gap(x) := \frac{L\_gap - x}{\mu 0\cdot A\_gap}$

$$F(Ni, x) := \frac{1}{2}\cdot\frac{2Ni^2}{\mu 0\cdot A\_gap\cdot(R\_core + R\_beam + 2\,R\_gap(x))^2}$$

$R\_core = 2.387\times 10^{8}$ $R\_beam = 7.958\times 10^{7}$ $R\_gap(0) = 1.989\times 10^{10}$

$R\_gap(5\cdot\mu m) = 9.947\times 10^{9}$

$Ni := 10 \qquad x := 5 \cdot \mu m$

Given $\quad F(Ni, x) = k \cdot x$

Pull-in Condition

$$\frac{d}{dx} F(Ni, x) = k$$

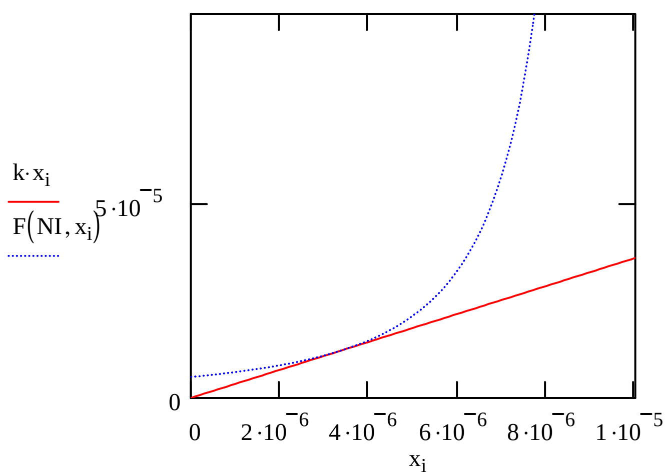
$$\begin{pmatrix} NI \\ x0 \end{pmatrix} := Find(Ni, x)$$

$NI = 2.098 \qquad x0 = 3.36 \times 10^{-6}$ Pull-in position

$I := \frac{NI}{100} \qquad I = 0.021$ Amp

$i := 0..100 \qquad x_i := \frac{i}{100} \cdot L\_gap$

Verification by
Graphic solution:



4)

(a) When the comb is rotated by an angle θ, the overlap area between a movable comb and a fixed comb can be approximated by $A(\theta) = \frac{1}{2}(L\theta + (L+a)\theta)a = \left(L + \frac{a}{2}\right)a\theta$, where L is the distance between the comb and the torsion beam, and *a* is the comb finger length, θ is the rotation angle.

$W^*(\theta) = 2N \frac{\varepsilon V^2}{2} \frac{A(\theta)}{g}$, where *g* is the finger gap spacing, and N is the number of movable comb fingers (the number of gap spacing is 2*N*). The electrostatic torque is $\frac{\partial W^*(\theta)}{\partial \theta} = 2N \frac{\varepsilon V^2}{2} \frac{(L + a/2)a}{g}$.

(b) $L := 1000 \cdot \mu m \quad a := 100 \cdot \mu m \quad g := 5 \cdot \mu m \quad w := 5 \cdot \mu m$

$$N := \frac{1000 \cdot \mu m}{2(w + g)} \qquad N = 50$$

$$\tau(V) := N \cdot \varepsilon 0 \cdot V^2 \cdot \frac{\left(L + \frac{a}{2}\right) \cdot a}{g}$$

$\tau(10) = 9.297 \times 10^{-10}$ N-m

(c) $\nu := 0.28 \quad L\_torsion := 500 \cdot \mu m \quad t := 50 \cdot \mu m$

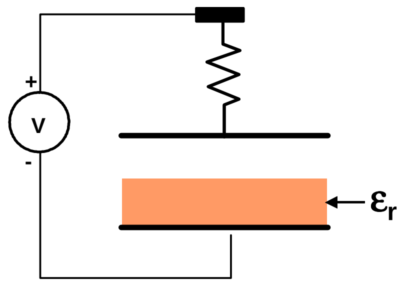
$$G := \frac{E}{2(1 + \nu)} \qquad G = 3.906 \times 10^{10}$$

$$K := \frac{t}{2} \cdot \left(\frac{w}{2}\right)^3 \cdot \left(\frac{16}{3} - 3.36 \cdot \frac{w}{t}\right)$$

$$\theta := \frac{\tau(10)}{2K \cdot \frac{G}{L\_torsion}}$$ $\theta = 3.048 \times 10^{-3}$ radian

(d) The angle can be increased by either torque or reduce the spring constant. The torque can be increased by reducing L (i.e., moving the combs closer to the torsion axis), increasing comb finger length a, reducing the gap spacing g, or increase the number of comb fingers. The spring constant is reduced by shrinking the width of the torsion beam (cubic dependence).

(2)



Assume the displacement is x. There is no simple expression for the capacitance, so we need to approach the problem from the energy density in each region.
Assume the electric field in the top region is E_1, and in the bottom region is E_2. They are related because the displacements D in both regions are continuous:

$$D_1 = \varepsilon_0 E_1 = D_2 = \varepsilon_0 \varepsilon_r E_2$$

$$\Rightarrow E_2 = \frac{1}{\varepsilon_r} E_1$$

Electrostatic energy density is $\frac{1}{2}\varepsilon E^2$, and the total electrostatic energy can be obtained by integrating the energy density over the volume:

$$W^* = \frac{\varepsilon_0}{2} E_1^2 A g_1 + \frac{\varepsilon_0 \varepsilon_r}{2} E_2^2 A g_2$$

$$= \frac{\varepsilon_0}{2} E_1^2 A g_1 + \frac{\varepsilon_0}{2} \frac{1}{\varepsilon_r} E_1^2 A g_2$$

Voltage, V, is equal to the integral of electric field along a field line between the two plates:

$$V = E_1 g_1 + E_2 g_2 = E_1 g_1 + E_1 \frac{1}{\varepsilon_r} g_2$$

$$E_1 = \frac{V}{g_1 + \frac{g_2}{\varepsilon_r}}$$

$$W^* = \frac{\varepsilon_0 V^2 A}{2} \frac{g_1}{\left(g_1 + \frac{g_2}{\varepsilon_r}\right)^2} + \frac{\varepsilon_0 V^2 A}{2} \frac{\frac{g_2}{\varepsilon_r}}{\left(g_1 + \frac{g_2}{\varepsilon_r}\right)^2}$$

$$W^* = \frac{\varepsilon_0 V^2}{2} \frac{A}{\left(g_1 + \frac{g_2}{\varepsilon_r}\right)} = \frac{\varepsilon_0 V^2}{2} \frac{A}{g_{eff}}$$

So it is similar to a gap closing actuator with an effective gap of $g_{eff} = \left(g_1 + \frac{g_2}{\varepsilon_r} \right)$.

$$g' = g_{eff} - x$$

$$F = -\frac{\partial W^*}{\partial x} = -\frac{\partial W^*}{\partial g'}\frac{\partial g'}{\partial x} = \frac{\partial W^*}{\partial g_{eff}} = \frac{\varepsilon_0 A V^2}{2}\frac{1}{g_{eff}^2}$$

$$g_1 = g_2 = \frac{g}{2}$$

$$F = \varepsilon_0 A V^2 \frac{2}{g^2}\frac{\varepsilon_r{}^2}{(\varepsilon_r + 1)^2}$$

Using the expression from lecture, the pull-in voltage would be

$$V_p = \sqrt{\frac{8kg_{eff}^3}{27\varepsilon_0 A}} = \sqrt{\frac{kg^3\left(\frac{\varepsilon_r + 1}{\varepsilon_r}\right)^3}{27\varepsilon_0 A}}$$

(b) The displacement at pull-in

$$d = \frac{g_{eff}}{3} = \frac{1}{3}\left(g_1 + \frac{g_2}{\varepsilon_r} \right) = \frac{g}{3}\left(\frac{\varepsilon_r + 1}{2\varepsilon_r} \right)$$

(c) Follow similar analysis, we can show that both the pull-in voltage and displacement remains the same as the first configuration.