EE 250B Midterm Solution Spring 2001

$\varepsilon0 := 8.854 \cdot 10^{-12}$ F/m $\quad um := 10^{-6}$

(1) (a) $N := 200 \quad V := 10 \quad t := 5 \cdot um \quad g := 1 \cdot um$

$$F(V) := N \frac{\varepsilon0 \cdot V^2}{2} \cdot \frac{t}{g}$$

$F(10) = 4.427 \times 10^{-7}$ N

(b) $L := 200 \cdot um \quad t := 5 \cdot um \quad w := 1 \cdot um \quad E := 160 \cdot 10^9$

$$I := \frac{t \cdot w^3}{12}$$

$$k := 4 \cdot \frac{6 \cdot E \cdot I}{L^3}$$

$k = 0.2$ N/m

(c)

$d := 10 \cdot um$

$V0 := 10$

Given

$F(V0) = k \cdot d \qquad k \cdot d = 2 \times 10^{-6}$

$V0 > 10$

$V := Find(V0) \qquad V = 21.255$

(2)
(a) Pull-in voltage of DMD:

$$V_{PI} = \sqrt{\frac{2 \cdot z_0^3}{\varepsilon_0 W L^3 C}} = \sqrt{\frac{2 \cdot z_0^3}{\varepsilon_0 W L^3} \left(\frac{16}{3} (\frac{w}{2})(\frac{t}{2})^3 \frac{E}{2(1+\nu)} \frac{1}{l} \right)}$$

Where z_0 is the mirror-electrode spacing, W is mirror width, L is half length of the mirror length, w, t, and l are the width, thickness, and length of the torsion spring, respectively, E is Young's modulus, ν is Poisson ratio. Assume all linear dimensions are proportion to a (= 1/10)

$$V_{PI} \propto \sqrt{\frac{a^3 \cdot a \cdot a^3}{a \cdot a^3 \cdot a}} = \sqrt{a^2} = a$$

The pull-in voltage reduces by 10 times too.

(b)

$$T = I \cdot B \cdot l_1 \cdot l_2 \cdot n = \theta / C = K \frac{G}{l} \theta$$

$$I = \left(\frac{16}{3} (\frac{w}{2})(\frac{t}{2})^3 \frac{E}{2(1+\nu)} \frac{1}{l} \right) \frac{\theta}{B \cdot l_1 \cdot l_2 \cdot n}$$

$$I \propto \frac{a \cdot a^3}{a} \frac{1}{a^2} = a$$

Therefore, the current also scales down by 10 times.

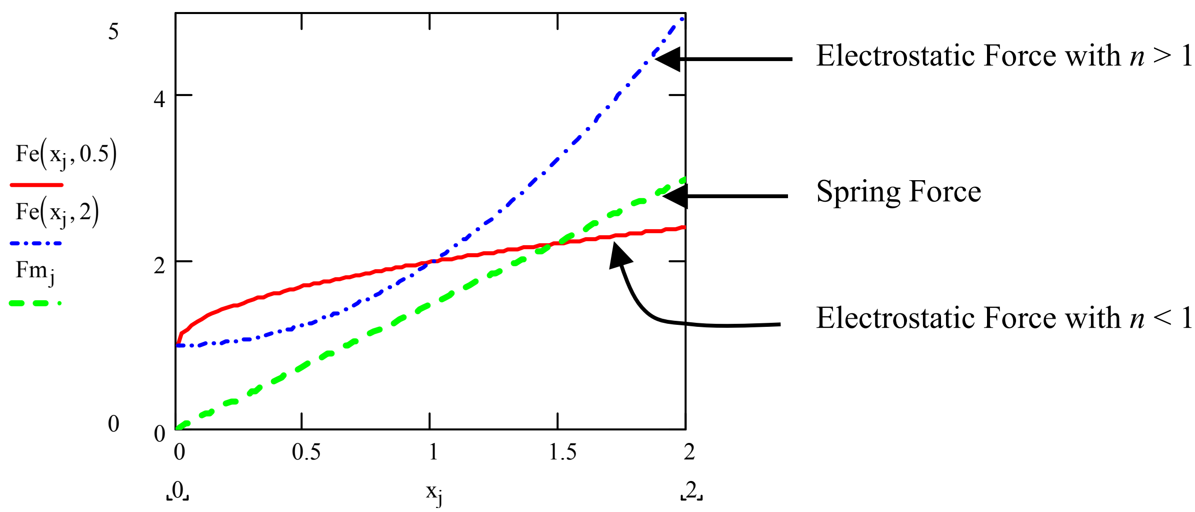
(3)

(a) Initially, the dielectric is just outside the parallel plate capacitor. For displacement = *x*, the electrostatic energy is

$$W^* = \frac{\varepsilon_0 V^2 W}{2} \frac{L-x}{d} + \int_0^x \frac{\varepsilon(x') V^2}{2} \frac{W}{d} dx' = \frac{\varepsilon_0 V^2 W}{2} \frac{L-x}{d} + \frac{\varepsilon_r V^2 W}{2d} \int_0^x (1 + a \cdot x'^n) dx'$$

$$F_e = \frac{\partial W^*}{\partial x} = -\frac{\varepsilon_0 V^2}{2} \frac{W}{d} + \frac{\varepsilon_r V^2 W}{2d} (1 + a \cdot x^n) = \frac{\varepsilon_0 V^2 W}{2d} \left(\frac{\varepsilon_r}{\varepsilon_0} (1 + a \cdot x^n) - 1 \right)$$

(b) Use graphic solution:

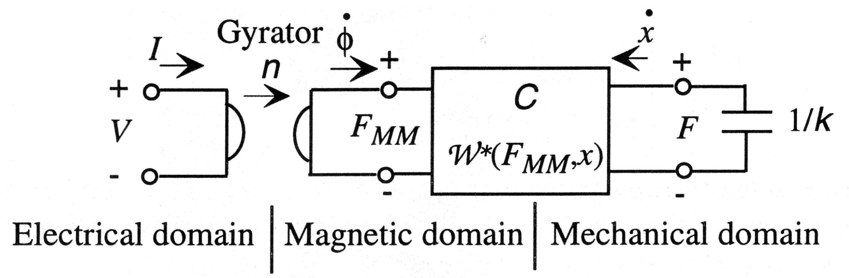


We can see that for *n* < 1, there is always an intersection between the electrostatic force curve and the spring force curve, no matter how large the voltage is. → Stable.
On the other hand, for *n* > 1, there is no intersection for sufficiently large voltage.
→ There is pull-in phenomena.

For *n* = 1, the electrostatic force curve is a straight line. The slope of the line is proportional to V^2. So there is always a solution for small V. For sufficiently large voltage, the electrostatic force curve moves up and there is no intersection. → There is pull-in phenomena.

(4)

(a) The lumped element model is



(b)

$$W^*(F_{MM},x)=\frac{F_{MM}^2}{2}\frac{1}{R}$$

$$F=\left.\frac{\partial W^*(F_{MM},x)}{\partial x}\right|_{F_{MM}}=\frac{F_{MM}^2}{2}\left(-\frac{1}{R^2}\frac{dR}{dg}\right)=-\frac{1}{2}\left(\frac{NI}{R}\right)^2\frac{dR}{dg}$$

where R is the reluctance, $F_{MM} = nI$ is the magnetomotive force, n is the number of coil turns, and I is the current, and g is the gap spacing of the cantilever. For magnetic core with large μ_{core} (= 1000), the reluctance is dominated by the reluctance of the gap:

$$R=\frac{g}{\mu_0 A}$$

$$F=-\frac{1}{2}\left(\frac{NI}{R}\right)^2\frac{dR}{dg}=-\frac{1}{2}\frac{N^2I^2}{g^2}\mu_0 A$$

(c)

$N := 100 \quad \mu0 := 4\cdot\pi\cdot 10^{-7} \quad A := 10\cdot um\cdot 10\cdot um \quad g0 := 10\cdot um$

$E := 100\cdot 10^{9} \quad W := 100\cdot um \quad H := 10\cdot um \quad L := 1000\cdot um$

Spring constant of cantilver $k := \frac{E\cdot W\cdot H^3}{4\cdot L^3}$ $\quad k = 2.5$

Magnetic Force: $Fm(I,g) := \frac{1}{2}\cdot\frac{N^2\cdot I^2}{g^2}\cdot\mu0\cdot A$

Spring Force $Fs(g) := k\cdot(g0 - g)$

Guess Initial Value: $I := 0.1 \quad g := 9\cdot um$

Given

$$Fm(I,g) = Fs(g)$$

$$\frac{d}{dg}Fm(I,g) = -k$$

$$\begin{pmatrix} I\_val \\ g\_val \end{pmatrix} := Find(I,g)$$

$I\_val = 0.024$

$g\_val = 6.667\times 10^{-6}$

Verify answer:

$Fm(I\_val, g\_val) = 8.333\times 10^{-6}$

$Fs(g\_val) = 8.333\times 10^{-6}$