A New Approach of

Blood Viscosity Measurement

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06/04/2001

Background

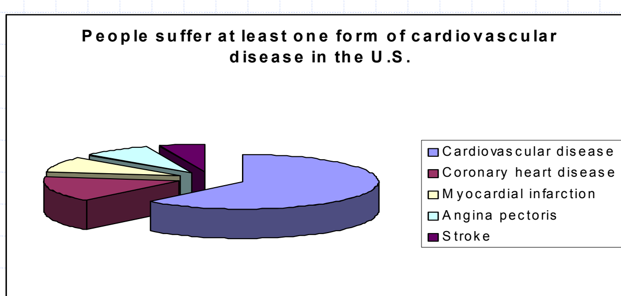
- The significance of viscosity of blood
  - Heart attacks
  - Strokes
  - Kidney diseases
  - Blindness
  - Aging

Viscosity Measurement

- A new method of measuring viscosity was proposed by Dr. Kensey, using capillary tube at different shear rates.
- Our approach
  - A consumable diagnostic tool
  - A cantilever beam is used.
  - Retain the blood-handling system.

Market Size

- 60 million people suffer at least one form of cardiovascular disease in the U.S.
- The total spending in the U.S. on diagnostic system in blood testing is expected to exceed $300 million.



Viscosity of Fluid

- Newton's Law Of Viscosity

$$\tau = \mu \frac{du}{dy}$$

where τ: shear force [N]

μ: dynamic viscosity [N*sec/m^2]

u: velocity of the fluid [m/sec]

Theories of Operation (1)

- Drag force per unit length:

$$F_y = F_d = \frac{1}{2}\rho |U_{rel}| U_{rel} D C_D$$

- Equation of Motion

$$m\ddot{y} + 2m\zeta_s \omega_n \dot{y} + ky = F_y = -\frac{1}{2}\rho |\dot{y}| \dot{y} D C_D$$

- Harmonic motion of the structure:

$$m\ddot{y} + 2m\omega_n[\zeta_s + \frac{2\rho D C_D A_y \omega}{3\pi m \omega_n}]\dot{y} + ky = 0$$

Theories of Operation (2)

- Fluid damping ratio ζ_f for a certain drag coefficient C_D:

$$\zeta_f = \frac{2}{3\pi}\frac{\rho D^2}{m}\frac{A_y}{D}\frac{\omega}{\omega_n}C_D$$

- Drag coefficient for certain fluid viscosity derived by Stokes in 1843 is:

$$C_D = \left(f\frac{D}{\dot{y}_{\max}}\right)\left(\frac{3\pi^2}{2}\right)\left(\frac{\mu}{\rho\pi f D^2}\right)^{\frac{1}{2}}$$

- Fluid damping coefficient can be approximated by

$$C_f = \left(4\pi^3 f D^2 \mu\rho\right)^{1/2}$$

Theories of Operation (3)

- Based on the fluid damping coefficient C_f, and neglect the structural damping, the peak vibration amplitude y_{max} is:

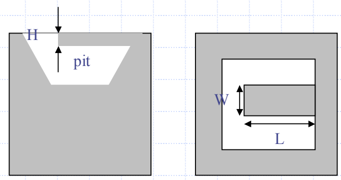
$$y_{\max} = \frac{Fl[1-\cos(\pi h/l)]/\pi}{(4\pi^3 f D^2 \mu\rho_l)^{1/2}\,\omega_0}$$

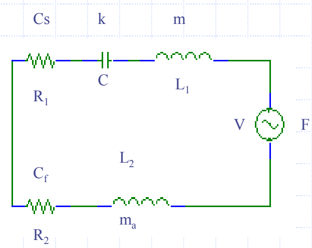
where F: driving force of the structure

h: length of the structure immersed in the fluid

l: length of the structure

Evaluation of Sensor's Performance





Electrical domain	Mechanical Domain
R_1	Cs
C	1/k
L_1	m
R_2	C_f
L_2	m_a

Resonant Frequency

Total impedance for this series circuit is

$$Zt= R+j\,X = (R_1 + R_2) + j\,(X_{L1} + X_{L2} - X_C)$$

Series resonance will occur when imaginary part equals to zero, thus,

$$(X_{L1} + X_{L2} - X_C) = 0,\ \omega L_1 + \omega L_2 - 1/(\omega C) = 0, \text{ therefore,}$$

$$\omega(L_1 + L_2) = 1/(\omega C),\ \omega^2 = 1/(L_1 + L_2)C$$

$$\omega = \sqrt{1/(L_1+L_2)C}$$

Recast into mechanical domain, so $\omega = \sqrt{k/(m+m_a)}$

For rectangle beam,

$$k = \frac{3EI}{L^3} = \frac{3E}{L^3} * \frac{WH^3}{12} = \frac{EWH^3}{4L^3}$$

We obtain the following, $\omega = \sqrt{\dfrac{EWH^3}{(m+m_a)L^3}}$

Quality Factor (1)

Q= (reactive power/ average power), as a result,

$$Q = \frac{I^2 X_L}{I^2 Z} = \frac{X_L}{R} = \frac{\varpi(L_1 + L_2)}{(R_1 + R_2)}$$

Again, in mechanical domain,

$$Q = \frac{\varpi(m + m_a)}{(C_s + C_v)} = \frac{\varpi}{\Delta\varpi}$$

Compare Q measured in fluid and atmosphere,

$$\frac{Q_f}{Q_a} = \frac{(m+m_a)}{(Cs+C_f)} / (\frac{m}{Cs}) = (\frac{m+m_a}{m})(\frac{Cs}{Cs+C_f})$$

Quality Factor (2)

$$\frac{\varpi_f}{\varpi_a} = \sqrt{\frac{m}{(m+m_a)}} = \frac{f_f}{f_a}$$,then $$\frac{Q_f}{Q_a} = (\frac{m+m_a}{m})(\frac{Cs}{Cs+C_f}) = \frac{f_a}{f_f} * \frac{1}{1 + \frac{C_f}{Cs}}$$

Furthermore,

$$Q = \frac{\varpi(m+m_a)}{(C_s + C_f)} = \sqrt{\frac{EWH^3}{(m+m_a)L^3}} \times \frac{(m+m_a)}{(C_s + C_f)} = \sqrt{\frac{EWH^3(m+m_a)}{L^3}} \times \frac{1}{(C_s + C_f)}$$

Because m_a is less than m, so we can ignore m_a and substitute m=ρV=ρHLW into the above eqn.

$$Q = \sqrt{\frac{EWH^3(\rho HLW)}{L^3}} \times \frac{1}{(C_s + C_f)} = \frac{WH^2}{L} \frac{\sqrt{\rho}}{(C_S + C_f)} \propto \frac{WH^2}{L}$$